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N63-15304
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TECHNICAL NOTE

D-1623

A PARAMETRIC INVESTIGATION OF THE LUNAR-ORBIT-RENDEZVOUS SCHEME

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Langley Station, Hampton, Va.

NATIONAL AERONAUTICS AND SPACE ADMINISTRATION

WASHINGTON April 1963

Code-1

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SUMMARY

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A parametric study of lunar-mission vehicles designed for lunar-orbit-rendezvous and direct lunar missions was made for the purpose of determining the injected weight required for missions performed under various circumstances. Missions were considered which had crew sizes from 2 to 14 men, transported supplies to be deposited on the moon up to 40,000 pounds, circular and elliptic orbits at the moon with maximum altitudes from 50 to 8,000 international nautical miles, and points of entry into lunar orbit at both apolune and perilune. Three fuel combinations were considered.

The results of this study indicate that the lunar-orbit-rendezvous mission requires much smaller weights injected to the moon than the direct lunar mission. For the lunar-orbit-rendezvous mission, the lowest lunar-mission-vehicle weights were generally obtained for low-altitude orbits. In the case of elliptic lunar orbits entered at perilune, vehicle weight was relatively insensitive to lunar-orbit altitude. In the cases of circular lunar orbits and elliptic lunar orbits entered at apolune, vehicle weight increased markedly with lunar-orbit altitude.

INTRODUCTION

In recent years the Langley Research Center has investigated the use of rendezvous to assist in accomplishment of the manned lunar mission. As a result of this work the merits of the use of rendezvous have become apparent, and a particular form of lunar mission has been developed which uses lunar orbit rendezvous. This mission substantially reduces the earth boost requirement for making a lunar mission. In this plan the command module in which the men make the trip to the moon and the associated propulsion for return to earth are left in a lunar orbit and descent to the lunar surface is made in a small lander vehicle. On return to the orbiting command module the lander vehicle is discarded and earth return is made in the command module which is designed for the required atmospheric reentry. As a result of avoiding the deceleration and acceleration of components not needed on the lunar surface the overall weight of the vehicle in transit to the moon is much less than would be required for a direct mission to the moon wherein all components are placed on the lunar surface. The substantial benefits of this lunar rendezvous concept were outlined in a summary of rendezvous research in reference 1 and to a further extent in reference 2.

The purpose of the present investigation was to study the lunar-orbit-rendezvous mission parametrically to determine the injected weight required for missions performed under various circumstances. In this regard, missions were considered which had:

(1) crew sizes ranging from 2 to 14 men,

(2) weights of transported supplies to be deposited on the moon of 0 and 40,000 pounds,

(3) maximum lunar-orbit altitudes from 50 to 8,000 international nautical miles,

(4) circular and elliptic lunar orbits with entry into and exit from the elliptic orbits made at apolune and perilune, and

(5) three different fuel combinations.

In addition, an analysis was made wherein the results were normalized in terms of the command-module weight in order to illustrate the relative effects of lander-capsule weight and weight transported to the moon. Throughout this report the direct lunar mission, wherein all components were taken to the lunar surface, is used for comparison.

SYMBOLS

E	total energy factor, $\frac{2U}{m}$, $(\text{ft/sec})^2$
g_e	acceleration of gravity at surface of earth, 32.2 ft/sec^2
g_m	acceleration of gravity at surface of moon, 5.32 ft/sec^2
H	total number of men in crew
h	altitude, international nautical miles
I	specific impulse, lb-sec/lb
K	mass-ratio factor, $K = \frac{MR}{1 + \left(k_T - k_G\right) - \left(k_T + k_C\right)MR} = \frac{W_i}{W_P}$

k	percentage weight factors $\left(\text{e.g., } k_G = \dfrac{\text{Weight of landing gear}}{\text{Weight supported by landing gear}}\right)$
MR	mass ratio, $\dfrac{W_i}{W_f}$
m	mass, slugs
r	radius, measured from center of lunar sphere, ft
r_m	radius of the lunar sphere, 5.702×10^6 ft
U	total energy, ft-lb
V	velocity, ft/sec
ΔV	change in velocity, ft/sec
W	weight, lb
α	pilotage factor, allowances made for deviations from the flight profiles used in the computations
β	the acute angle between the earth-moon line and the asymptote of a hyperbolic lunar orbit, deg
γ	flight-path angle, angle made by velocity vector with local lunar horizontal, deg
ϵ	orbital eccentricity
θ	orbital angle measured from perilune, deg

Subscripts:

a,b,c,d	quantities associated with four propulsive efforts of lunar-orbital-rendezvous mission
e,f	quantities associated with four propulsive efforts of direct lunar mission
a	apolune
B	supplies container
C	circular, when referring to velocities; thrust and attitude controls when referring to weights

DLV	direct lunar vehicle
E	elliptic
F	fuel
f	final
G	landing gear
H	hyperbolic when referring to orbital elements; man when referring to weights
i	initial
L	lunar-lander manned module including lander crew (i.e., one less than total crew)
LORV	lunar-orbital-rendezvous vehicle
M	command module including total crew
m	surface of moon
max	maximum altitude
P	payload
p	perilune
R	rotation of elliptic lunar orbit with respect to earth-moon line, used in appendix A
S	supplies
T	tanks and engines
α	apolune of Hohmann descent ellipse when used in section "Propulsive Increments"
π	perilune of Hohmann descent ellipse when used in section "Propulsive Increments"
50	altitude of 50 nautical miles

Vehicle designations:

DLV	direct lunar vehicle
L	lunar-lander manned module
LLV	lunar-lander vehicle

LORV lunar-orbit-rendezvous vehicle

M command module (crew capsule)

S transported supplies

MISSION PROFILE

The mission profile for the lunar-orbit-rendezvous mission considered in this investigation is shown in figure 1. A similar profile is shown for the direct lunar mission in figure 2. The operations of most significance in this study are establishment of lunar orbit, descent to surface with lander vehicle, take-off for lunar rendezvous with command module left in orbit, and orbital launch for earth return in command module. Although specific allowance was not made for a plane change at the moon this situation is considered to be adequately covered by a percentage allowance for deviation from the profiles given here.

Three lunar-orbit situations were assumed for the investigation. (See fig. 3.) In one situation, circular lunar orbits of various altitudes were considered. In the other two situations, elliptic orbits having various maximum altitudes and a perilune distance of 50 nautical miles were considered. For elliptic orbits, in one case, entrance and exit from lunar orbit were made at perilune; in the other case, at apolune. It is recognized that stay time and the initial inclination of the lunar orbit, in general, will dictate the point in lunar orbit for injection to earth return and will prohibit operation exactly from either apolune or perilune, but these conditions were chosen as representative of the situations that will be faced in orbit establishment. Appendixes A and B give a more careful examination of this matter in terms of the direction of approach and departure from the moon.

In this investigation, descent to the lunar surface and launch to lunar rendezvous with the command module are assumed to be accomplished by a Hohmann transfer. It is recognized that, in general, shorter transfers may be more practical from guidance, control, and other considerations, but for assessment of relative weights the Hohmann transfer was believed to be adequate. In this regard, one of the more attractive descent orbits is one having a period equal to that of the rendezvous orbit. In this case, rendezvous 1 period later is facilitated in the event that final braking and descent is deferred. A substantial allowance was made to account for such deviations from the Hohmann transfer.

For the purpose of establishing velocity increments, the sequence of orbits in the direct lunar mission was assumed to be the same as for the lunar-orbit-rendezvous missions. In the direct lunar mission, the entire lunar vehicle was taken to the surface of the moon.

The impulsive velocity increments necessary to obtain the various trajectories considered in this investigation are given in table I. Velocity increments ΔV_a, ΔV_b, ΔV_c, and ΔV_d apply to the lunar-orbit-rendezvous mission. Velocity increments ΔV_a and ΔV_d are required for braking into lunar orbit and injection to earth return. Velocity increments ΔV_b and ΔV_c are required for landing on the moon and launch to rendezvous in lunar orbit. Velocity increments ΔV_e and ΔV_f apply to the direct lunar mission and are required for braking and landing on the moon and launch and injection to earth return, respectively.

These velocity increments were multiplied by the factors indicated in table II to allow for orbital plane changes, gravity influence due to finite thrusting times, and piloting errors. The method of utilizing these velocity increments to calculate the vehicle weights for the conditions investigated is discussed in "Method of Analysis."

LUNAR-MISSION VEHICLES

Lunar-Orbit-Rendezvous Vehicle

A schematic of the lunar-orbit-rendezvous vehicle considered is shown in figure 4. This vehicle consists of a command module M, propulsive elements a and d, and a lunar lander L, c, S, and b. The propulsive element a serves to brake the entire vehicle into lunar orbit, and the propulsive element d, to inject the command module M to earth return. The lander vehicle has propulsive elements b and c, a supply element S, and a manned module L. The propulsive element b brakes the lander to the surface of the moon, and the propulsive element c launches the manned module L to a lunar rendezvous with the command module M.

A significant version of the lunar-orbit-rendezvous vehicle is obtained if the propulsive element d is omitted. Propulsive element a is then used to brake the lander vehicle and command module into lunar orbit and to launch the command module to earth return. This plan is reasonable if no large supply weights are deposited on the moon in that the velocity increment associated with braking into and launch from lunar orbit is only a total of about 6,600 ft/sec. Staging boosters at velocity increments of 10,000 ft/sec or more is accepted as good practice. In this investigation it was intended to study the effect of transporting large weights to the lunar surface and the booster requirements for this task are inconsistent with the requirements for launch of the command module to earth return; therefore, staging was employed to obtain a more realistic weight structure.

For purposes of this analysis, the fuel-tank weight was assumed to be proportional to the fuel contained so that $W_T = k_T W_F$. The attitude control system of a given stage was assumed to be proportional to the stage initial weight so that $W_C = k_C W_i$. The landing gear was assumed to be proportional to stage final weight so that $W_G = k_G W_f$. The factors k_T, k_C, and k_G are shown in figure 4 for the various propulsive efforts. For propulsive efforts a, c, and d, k_G is 0 because no landing gear is necessary on these stages.

The command-module weight was considered to be a function of the mission crew size. The weights for the various crew sizes included in this investigation are given in table III. The items that make up these weights are a fixed weight of 1,000 pounds for instruments, guidance, and communications; a weight of 2,375 pounds per man for men and associated equipment; a structural weight equal to 0.25 of the first two items; and a heat shield weight equal to 1,300 $(H/3)^{2/3}$.

The lander-module (L) weight was considered to be a function of lander crew size. The weights considered for the various crew sizes included in this investigation are given in table IV. In all cases, the lander crew is considered to be one less than the mission crew (H - 1). One man is left in charge of the command module on descent to the moon. The weight of the lander module is constituted of a fixed weight of 535 pounds for guidance, instrumentation, and communication; a weight of 439 pounds per man for a man, life support, and associated gear; and a structural weight of 0.25 of the sum of the first two items.

The weight of the container for the supplies to be transported to the moon was assumed to be proportional to the supply weight so that $W_B = k_S W_S$. The factor k_S was taken to be 0.25. A man and space suit were assumed to weigh 200 pounds.

For comparison, a single-stage lunar lander was considered. This vehicle is shown schematically in figure 5. Propulsive elements b and c are employed as for the two-stage lunar lander, but in this case the fuels are contained in a single tank. The weights of lander module L, fuel tank, control system, landing gear, and supply container were defined in much the same way as was employed for the two-stage lunar lander. The fuel-tank weight was assumed to be proportional to the fuel contained so that $W_T = k_T\left(W_{F,b} + W_{F,c}\right)$; the attitude-control-system weight was assumed to be proportional to the initial weight of the vehicle so that $W_C = k_C W_{i,b}$; the landing-gear weight was assumed to be proportional to the weight of the vehicle landed on the moon so that $W_G = k_G W_{f,b}$; and the supply-container weight was assumed to be proportional to the weight of the supplies so that $W_B = k_S W_S$. The values of the factors k_T, k_C, k_G, and k_S employed for these calculations are given in figure 5.

Direct-Lunar-Mission Vehicle

A schematic of the direct-lunar-mission vehicle considered is shown in figure 6. This vehicle consists of a command module M, transported supplies S, and propulsive elements e and f. The propulsive element e serves to brake and land the entire vehicle at the moon, and the propulsive element f serves to launch and inject the command module M to earth return. The considerations concerning the weights of fuel tank, the control system, and the landing gear were much the same for this vehicle as for the lunar-orbit-rendezvous vehicle. The weight factors for the two propulsive efforts e and f are given in figure 6.

Fuel Combinations

Two fuels were considered in this investigation. One was hydrogen/oxygen with a specific impulse of 425 seconds; the other was nitrogen tetroxide/unsymmetrical dimethyl hydrazine with a specific impulse of 315 seconds. These fuels were considered in the combinations shown in table V for the various phases of the lunar missions studied. Fuel combination 2 (425/315) involved the use of the fuel with specific impulse of 315 in the lander and the fuel with specific impulse of 425 for braking into and launch from lunar orbit. This combination was not considered for the direct lunar mission.

METHOD OF ANALYSIS

Unit Rocket Equation

Consider a rocket which consists of a useful payload, a landing gear, attitude control system, tanks and engines, and a fuel supply. (See fig. 7.) The initial weight of such a rocket may be expressed as the sum of these components as follows:

$$W_i = W_P + W_G + W_C + W_T + W_F \tag{1}$$

The final weights after a propulsive effort which consumes the fuel may be written as:

$$W_f = W_i - W_F$$

which, for later convenience, may be written

$$W_F = W_i - W_f \tag{2}$$

Now the landing gear, attitude control, and tank and engine weights may be written as simple proportions of their governing weights (i.e., final, initial, and fuel weights, respectively) so that

$$\left.\begin{aligned} W_G &= k_G W_f \\ W_C &= k_C W_i \\ W_T &= k_T W_F \end{aligned}\right\} \tag{3}$$

Substituting equations (3) into equation (1) gives

$$W_i = W_P + k_G W_f + k_C W_i + k_T W_F + W_F \tag{4}$$

Equation (4) reduces to the following equation:

$$\left(1 - k_C\right) W_i = W_P + k_G W_f + \left(1 + k_T\right) W_F \tag{5}$$

Substituting equation (2) into equation (5) results in

$$\left(1 - k_C\right) W_i = W_P + k_G W_f + \left(1 + k_T\right)\left(W_i - W_f\right) \tag{6}$$

Now substituting $W_f = \frac{W_i}{MR}$ for the final weight and combining terms gives

$$\left[\frac{1 + \left(k_T - k_G\right)}{MR} - \left(k_T + k_C\right)\right] W_i = W_P$$

and dividing by the quantity inside the brackets gives the following result:

$$W_i = \frac{W_P MR}{1 + \left(k_T - k_G\right) - \left(k_T + k_C\right) MR} \tag{7}$$

Equation (7) may be written as

$$W_i = W_P K \tag{8}$$

where

$$K = \frac{MR}{1 + \left(k_T - k_G\right) - \left(k_T + k_C\right) MR} \tag{9}$$

and the mass ratio may be written as a function of the change in velocity resulting from the propulsive effort as follows:

$$MR = e^{\frac{\Delta V \alpha}{g_e I}} \tag{10}$$

where the factor α accounts for the influence of gravity during the finite burning time, plane changes, and piloting inefficiency. (See table II.)

Lunar-Orbit-Rendezvous Rocket Equation

Consider the entire lunar-orbit-rendezvous-mission vehicle. (See fig. 4.) The vehicle shown is staged after each propulsive effort because of the large masses transported in some missions considered. When a large mass is deposited on the lunar surface only a modest thrust capability is required to either return the small lander capsule to orbit or inject the command module to earth return in proportion to that required initially to establish orbit or to land. In cases involving more or less constant payloads, staging for velocity increments less than 10,000 feet per second could hardly be justified because of the additional complexity involved.

The initial weight of the entire lunar-orbit-rendezvous vehicle is formulated by combining the unit rocket equation (eq. (8)) appropriately for the vehicle elements of figure 4. In this formulation the payload element W_P of the unit rocket equation has different values for the various propulsive efforts. These values may be obtained by summing the elements of figure 4, and are

$$\left.\begin{aligned} W_{P,a} &= W_{i,d} + W_{i,b} - (H - 1)W_H \\ W_{P,b} &= W_{i,c} + (1 + k_S)W_S \\ W_{P,c} &= W_L \\ W_{P,d} &= W_M \end{aligned}\right\} \tag{11}$$

By use of the unit rocket equation (eq. (8)), the following equations are obtained:

$$W_{i,a} = W_{P,a}K_a \tag{12}$$

and

$$\left.\begin{aligned} W_{i,b} &= W_{P,b}K_b \\ W_{i,c} &= W_{P,c}K_c \\ W_{i,d} &= W_{P,d}K_d \end{aligned}\right\} \tag{13}$$

Substituting equations (11) and (13) into equation (12) gives the following equation for the initial weight of the vehicle in transit to the moon:

$$W_{i,a} = \left\{W_M K_d + \left[W_L K_c + (1 + k_S)W_S\right]K_b - (H - 1)W_H\right\} K_a$$

and finally when normalized with respect to the command-module weight

$$\frac{W_{i,a}}{W_M} = \left\{K_d + \left[\frac{W_L}{W_M} K_c + (1 + k_S)\frac{W_S}{W_M}\right]K_b - (H - 1)\frac{W_H}{W_M}\right\} K_a \tag{14}$$

The mass-ratio factors K_a; K_b, K_c, and K_d correspond to propulsive increments ΔV_a, ΔV_b, ΔV_c, and ΔV_d, respectively. (See eqs. (9) and (10).) The factor k_S when multiplied by the weight of the transported supplies gives the weight of the containing structure. This factor was taken as 0.25 in this analysis. The factor W_H is the weight of one man and a space suit, and $(H - 1)$ is the number of men carried in the lander vehicle.

If two lander vehicles are carried on the mission, then equation (14) becomes

$$\frac{W_{i,a}}{W_M} = \left\{K_d + 2\left[\frac{W_L}{W_M} K_c + (1 + k_S)\frac{W_S}{W_M}\right]K_b - (H - 1)\frac{W_H}{W_M}\right\} K_a$$

Direct-Lunar-Mission Rocket Equation

Consider now the entire direct-lunar-mission vehicle. (See fig. 6.) In this case,

$$\left.\begin{aligned} W_{P,e} &= W_{i,f} + (1 + k_S)W_S \\ W_{P,f} &= W_M \end{aligned}\right\} \tag{15}$$

and, from the unit rocket equation (eq. (8)),

$$W_{i,e} = W_{P,e}K_e \tag{16}$$

and

$$W_{i,f} = W_{P,f}K_f \tag{17}$$

Substituting equations (15) and (17) into equation (16) gives the following equation for the initial weight of the direct-lunar-mission vehicle in transit to the moon:

$$W_{i,e} = \left[W_M K_f + \left(1 + k_S\right)W_S\right]K_e$$

and finally when normalized with respect to the command-module weight

$$\frac{W_{i,e}}{W_M} = \left[K_f + \left(1 + k_S\right)\frac{W_S}{W_M}\right]K_e \tag{18}$$

The mass-ratio factors K_e and K_f correspond to propulsive increments ΔV_e and ΔV_f, respectively. (See eqs. (9) and (10).)

The ratio of the injected weight for a lunar-orbit-rendezvous mission in comparison with that for a direct mission is the ratio of equation (14) to equation (18).

$$\frac{W_{i,LORV}}{W_{i,DLV}} = \frac{K_a\left\{K_d + \left[\frac{W_L}{W_M} K_c + \left(1 + k_S\right)\frac{W_S}{W_M}\right]K_b - (H - 1)\frac{W_H}{W_M}\right\}}{\left[K_f + \left(1 + k_S\right)\frac{W_S}{W_M}\right]K_e} \tag{19}$$

For a parametric analysis consider a three-man mission such that $(H - 1) = 2$ and $\frac{W_H}{W_M} = 0.0175$ then

$$\frac{W_{i,LORV}}{W_{i,DLV}} = \frac{K_a \left\{K_d + \left[\frac{W_L}{W_M} K_c + (1 + k_S)\frac{W_S}{W_M}\right]K_b - 0.0350\right\}}{\left[K_f + (1 + k_S)\frac{W_S}{W_M}\right]K_e} \tag{20}$$

Single-Stage Lander Rocket Equation

Consider the case of a single-stage lander vehicle. (See fig. 5.) In this case there is no staging of tanks on the moon; however, there is allowance for the deposit of supplies after landing. The propulsive efforts are indicated as b and c corresponding to the propulsive efforts of the two-stage lander vehicle shown in figure 4. These efforts correspond to landing on the moon and take-off, respectively.

The weights of the tank, control system, landing gear, and supply container are defined as

$$\left.\begin{aligned} W_T &= k_T\left(W_{F,b} + W_{F,c}\right) \\ W_C &= k_C W_{i,b} \\ W_G &= k_G W_{f,b} \\ W_B &= k_S W_S \end{aligned}\right\} \tag{21}$$

so that the total final weight of the single-stage lander may be written as

$$W_{f,c} = W_L + W_C + W_G + W_T \tag{22}$$

where $W_{F,b}$ and $W_{F,c}$ refer to weights of fuel for propulsive efforts b and c, $W_{i,b}$ refers to the initial weight of the lander prior to propulsive effort b, $W_{f,b}$ refers to the final weight of the lander after propulsive effort b, W_S refers to the weight of supplies transported to the moon, and W_L refers to the weight of the lander capsule. Now the mass ratio becomes

$$MR_b = \frac{W_{i,b}}{W_{f,b}} \tag{23}$$

and

$$MR_c = \frac{W_{i,c}}{W_{f,c}} \tag{24}$$

Because of the deposit of supplies,

$$W_{f,b} - \left(1 + k_S\right)W_S = W_{i,c} \tag{25}$$

Combining equations (23), (24), and (25) gives

$$W_{i,b} = MR_b\left[MR_c W_{f,c} + \left(1 + k_S\right)W_S\right] \tag{26}$$

Also,

$$W_{F,c} = \left(MR_c - 1\right)W_{f,c} \tag{27}$$

and

$$W_{F,b} = \left(MR_b - 1\right)\left[MR_c W_{f,c} + \left(1 + k_S\right)W_S\right] \tag{28}$$

Substituting equations (21), (22), (23), (24), (25), (27), and (28) into equation (26) and solving for $W_{i,b}$ gives the following equation for the initial weight of a single-stage lander:

$$W_{i,b} = \left\{\frac{MR_c W_L + \left[1 - k_T\left(MR_c - 1\right)\right]\left(1 + k_S\right)W_S}{1 - k_G MR_c - k_C MR_b MR_c - k_T\left(MR_b MR_c - 1\right)}\right\} MR_b \tag{29}$$

where $MR = e^{\frac{\Delta V \alpha}{g_e I}}$. Equation (29) may be combined with the unit rocket equation (eq. (8)) for propulsive efforts a and d of the vehicle shown in figure 4 to obtain the initial weight of a lunar-orbit-rendezvous vehicle having a single-stage lander. In this case,

$$W_{i,d} = W_M K_d$$

$$W_{i,b} = W_{i,lander} \qquad \text{(from eq. (29))}$$

$$W_{i,a} = \left[W_M K_d + W_{i,lander} - (H - 1)W_H\right]K_a$$

and finally

$$\frac{W_{i,a}}{W_M} = K_a\left[K_d + \frac{W_{i,lander}}{W_M} - (H - 1)\frac{W_H}{W_M}\right]$$

Propulsive Increments

The velocity increments necessary for accomplishment of the lunar-orbit-rendezvous mission are given as ΔV_a, ΔV_b, ΔV_c, and ΔV_d in table I. These increments are the impulsive values required for accomplishing the required orbital transfers according to two-body theory. The velocity increments ΔV_e and ΔV_f are those required for the direct lunar mission. These quantities were calculated from the following formulation.

Lunar-orbit-rendezvous mission.- For a circular lunar orbit, the following velocity increments are used:

For entrance into lunar orbit,

$$\Delta V_a = V_H - V_C$$

for descent and landing on the moon,

$$\Delta V_b = \left(V_C - V_\alpha\right) + V_\pi$$

for ascent to lunar orbit,

$$\Delta V_c = \left(V_C - V_\alpha\right) + V_\pi$$

and, for launch out of lunar orbit to an earth return,

$$\Delta V_d = V_H - V_C$$

For an elliptic lunar orbit entered at apolune, the following velocity increments are used:

For entrance into lunar orbit,

$$\Delta V_a = V_H - V_a$$

for descent and landing on the moon,

$$\Delta V_b = \left(V_p - V_\alpha\right) + V_\pi$$

for ascent to lunar orbit,

$$\Delta V_c = \left(V_p - V_\alpha\right) + V_\pi$$

and, for launch out of lunar orbit to an earth return,

$$\Delta V_d = V_H - V_a$$

For an elliptic lunar orbit entered at perilune, the following velocity increments are used:

For entrance into lunar orbit,

$$\Delta V_a = V_{H,50} - V_p$$

for descent and landing on the moon,

$$\Delta V_b = \left(V_p - V_\alpha\right) + V_\pi$$

for ascent to lunar orbit,

$$\Delta V_c = \left(V_p - V_\alpha\right) + V_\pi$$

and, for launch out of lunar orbit to an earth return,

$$\Delta V_d = V_{H,50} - V_p$$

Direct lunar mission.- For direct lunar missions corresponding to each of the three modes of lander missions, the following velocity increments are used:

For braking, descent, and landing,

$$\Delta V_e = \Delta V_a + \Delta V_b$$

and, for ascent to orbit and launch,

$$\Delta V_f = \Delta V_c + \Delta V_d$$

The velocities required for these expressions are obtained from two-body theory with $V_{H,50} = 8{,}700$ ft/sec given to establish a reasonable energy level for the hyperbolic lunar approach trajectories.

The hyperbolic velocities are

$$V_H = \left(E_H + 2V_C^2\right)^{1/2}$$

where the total hyperbolic energy factor E_H is

$$E_H = V_{H,50}^2 - 2V_{C,50}^2$$

The circular satellite velocities are

$$V_C = \left(\frac{r_m}{r_{max}}\right)^{1/2} V_{C,m}$$

$$V_{C,50} = \left(\frac{r_m}{r_{50}}\right)^{1/2} V_{C,m}$$

where the circular satellite velocity at the surface of the moon $V_{C,m}$ is obtained from the expression

$$V_{C,m} = \left(g_m r_m\right)^{1/2}$$

The elliptic lunar orbit satellite velocities are apolune velocity

$$V_a = 2^{1/2}\left(\frac{r_m}{r_{max} + r_{50}}\right)^{1/2}\left(\frac{r_{50}}{r_{max}}\right)^{1/2} V_{C,m}$$

and perilune velocity

$$V_p = 2^{1/2}\left(\frac{r_m}{r_{max} + r_{50}}\right)^{1/2}\left(\frac{r_{max}}{r_{50}}\right)^{1/2} V_{C,m}$$

The Hohmann descent velocities are apolune (initiation of descent) velocity

$$V_\alpha = 2^{1/2}\left(\frac{r_m}{r + r_m}\right)^{1/2}\left(\frac{r_m}{r}\right)^{1/2} V_{C,m}$$

and perilune (touchdown) velocity

$$V_\pi = 2^{1/2}\left(\frac{r_m}{r + r_m}\right)^{1/2}\left(\frac{r}{r_m}\right)^{1/2} V_{C,m}$$

where r in the equations for V_α and V_π takes the value of r_{max} for descent from a circular orbit and r_{50} for descent from an elliptic orbit.

RESULTS

The results of the calculation of vehicle weights for the lunar-orbit-rendezvous and direct lunar missions considered in this investigation are given in table VI. This table gives the entire lunar-vehicle weight approaching the moon and lunar-lander-vehicle initial weight for the lunar-orbit-rendezvous missions and the entire lunar-vehicle weight approaching the moon for the direct lunar missions. Values are given for the specific-impulse combinations of table V, for various orbit altitudes, for both circular and elliptic lunar orbits, for entrance into elliptic orbits at both apolune and perilune, and for weights transported to the moon of 0 and 40,000 pounds. Some of these results are plotted in figures 8 to 18 in order to better illustrate the effects involved. Figures 8 to 13 show the effects of orbit altitude and specific impulse on vehicle weights for three-man lunar missions with circular lunar orbits and elliptic lunar orbits entered at apolune and perilune. Figures 14 to 18 show the effects of transported weight and mission complement on vehicle weights for lunar missions with close circular lunar orbits (h = 100 nautical miles) and three specific-impulse combinations. Figures 19 and 20 give a comparison of the weights of lunar-orbit-rendezvous- and direct-lunar-mission vehicles as a function of transported weight for two specific impulses. These results are for three-man crews and circular lunar orbits with altitude of 100 nautical miles. Figure 21 shows the effect of varying the ratio of module weights (command to lunar lander) on the ratio of vehicle weights (lunar orbit rendezvous to direct mission) for various amounts of weight transported to the moon. Table VII gives a comparison of the initial weights of one-stage and two-stage lunar-lander vehicles. The two-stage vehicle was used for most of this investigation.

DISCUSSION

Effect of Orbit Altitude

The substantial weight advantage of the lunar-orbit-rendezvous mission in comparison with the direct lunar mission is readily evident on examination of the results of table VI. The lunar-orbit-rendezvous mission requires much less vehicle weight for all the missions considered. For no transported weight the ratio of vehicle weights (lunar-orbit-rendezvous mission to direct lunar mission) is 1/3 or less. Lunar-orbit altitude has a substantial effect on the weights of lunar vehicles for both the lunar-orbit-rendezvous and direct lunar missions in a majority of the cases investigated. Vehicle weights increase with orbit altitude for circular lunar orbits and elliptic lunar orbits entered at apolune. The weight of the direct-lunar-mission vehicle is not affected by lunar-orbit altitude for the elliptic lunar orbit entered at perilune. (See figs. 10 and 13 and table VI.) The insensitivity to lunar-orbit altitude in this case results from the fact that the velocity increments do not change with lunar-orbit altitude. (See table I.)

The weight of the lunar-orbit-rendezvous vehicle is affected by lunar-orbit altitude in varying ways for the case of the elliptic lunar orbit entered at perilune depending on the transported weight and specific-impulse combination employed. (See figs. 8 and 11.) When a supply package of 40,000 pounds is transported to the moon the vehicle weights increase appreciably with orbit altitude for all specific-impulse combinations investigated. (See fig. 11.) In figure 8, when no weight is transported to the moon the effect of orbit-altitude change is dependent on the specific-impulse combination chosen. For a mission with a specific impulse of 315 throughout, the minimum vehicle weight occurs at about 750 nautical miles. For a mission with a specific impulse of 315 employed in the lander and a specific impulse of 425 employed for deceleration into and launch from lunar orbit a different result is obtained. In this case vehicle weight increases with orbital altitude throughout the range studied. (See fig. 8.) For a mission with a specific impulse of 425 throughout, the vehicle weight decreases with increase in orbital altitude. The major decrease in vehicle weight is obtained for an increase in orbital altitude to 2,000 nautical miles. Little additional benefit accrues when the maximum orbital altitude is increased to 8,000 nautical miles. Basically the changes in vehicle weight with orbital altitude for the elliptic orbit entered at perilune are small in comparison with the changes that occur for the other two types of lunar orbits considered.

The weights of the lunar landers which descend from the perilune of the elliptic lunar orbits are appreciably lighter than those of the landers which descend from the circular lunar orbit. The velocity increment required for descent to the lunar surface from a circular lunar orbit is greater than that required for descent from an elliptic orbit of the same maximum altitude. This difference requires a greater propulsive weight for the lander in circular orbit. (See table VI.)

Effect of Transported Weight

Transporting cargo to the lunar surface and increasing the crew size increases the weight of the required lunar vehicle. (See table VI and figs. 14 to 18.) A comparison of vehicle weights for direct and lunar-orbit-rendezvous vehicles as conceived for this study is given in figures 19 and 20 for a three-man mission using a circular lunar orbit with altitude of 100 nautical miles. The rate of change of vehicle weight with increase in transported weight is only slightly different for the two mission concepts. As greater weights are transported the direct-lunar-mission-vehicle weight becomes closer percentagewise to the weight of the lunar-orbit-rendezvous vehicle. With a transported weight of 40,000 pounds, however, the three-man direct mission vehicle is still 1.83 times as heavy as the lunar-orbit-rendezvous-mission vehicle for a specific impulse of 315 seconds. For a specific impulse of 425 seconds this ratio is about 1.35. For a specific impulse of 315 and 425 seconds and no weight transported to the moon, this ratio is 5.35 and 3.08, respectively.

Effect of Lander Weight

Changes in the ratio of lander-capsule weight to command-module weight as would be required in order to change the environmental situation for the lander crew has a substantial effect on the relative weights of lunar-orbit-rendezvous and direct-lunar-mission vehicles. (See fig. 21.) The range of the ratio of lander-capsule weight to command-module weight used in most of this investigation is indicated to be about 0.16. Varying this factor from 0 to 0.4 changes the ratio of lunar-orbit-rendezvous-vehicle weight to direct-lunar-mission-vehicle weight from about 0.2 to about 0.5 for no transported weight. As the transported weight is increased the sensitivity of this ratio to lander-capsule weight is substantially decreased. In these calculations the lander is assumed to always carry two men to and from the moon even when the lander-capsule weight goes to 0. This assumption was felt to be reasonable in that the purpose of the calculation was to illustrate the effect of different design concepts for the lander module. In some cases, simple unenclosed designs have been proposed which weigh very little. In other cases more substantial "shirt-sleeve" environment designs have been put forward.

<u>Effect of staging on lunar-lander weight</u>.- A two-stage lunar lander is appreciably lighter than a single-stage lunar lander for the conditions

investigated. (See table VII.) However, when no weight was transferred to the lunar surface and the specific impulse of the fuel was 425 seconds the weight penalty for the use of a single-stage lander was only 25 percent. Where 40,000 pounds of supplies were deposited on the moon and a specific impulse of 315 seconds was employed, the single-stage lander weighed about three times as much as the two-stage lander.

CONCLUDING REMARKS

A parametric study of lunar-mission vehicles designed for lunar-orbit-rendezvous and direct lunar missions was made for the purpose of determining the injected weight required for missions performed under various circumstances.

Weights for vehicles in transit to the moon were obtained for missions which had crew sizes from 2 to 14 men, transported supplies to be deposited on the moon up to 40,000 pounds, circular and elliptic orbits at the moon with maximum altitudes from 50 to 8,000 nautical miles, points of entry into elliptic lunar orbit at both apolune and perilune, and three fuel combinations.

The vehicle weight in transit to the moon was much less for the lunar-orbit-rendezvous missions than for the direct lunar missions. For the cases where no weight was transported to be left on lunar surface, the ratio of injected weights varied from about 0.4 to 0.1 depending on the fuel combination and lunar-orbit altitude considered.

For the lunar-orbit-rendezvous mission the lowest lunar-mission-vehicle weights were generally obtained for low-altitude orbits. For elliptic lunar orbits entered at perilune, vehicle weight was relatively insensitive to lunar-orbit altitudes. For circular lunar orbits and elliptic lunar orbits entered at apolune, vehicle weight increased markedly with lunar-orbit altitude.

For a booster with an injection capability of 120,000 pounds, the direct three-man lunar mission, as analyzed herein, using fuel with a specific impulse of 425 seconds would have no capability for transporting supplies to be left on the moon. The comparable lunar-orbit-rendezvous mission would have the capability of transporting about 20,000 pounds of supplies or scientific equipment to the moon.

Langley Research Center,
National Aeronautics and Space Administration,
Langley Station, Hampton, Va., January 14, 1963.

APPENDIX A

ESTABLISHMENT OF ELLIPTIC LUNAR ORBITS

Consider the problem of the establishment of an elliptic lunar orbit with the major axis alined in a chosen direction with respect to the earth-moon line. The orbit to be established at the moon and the transfer orbit to the moon are assumed to be coplanar. Figure 22 shows the geometry of the problem. The angle θ_R through which the major axis of the elliptic lunar orbit is rotated with respect to the earth-moon line is specified. Also, the elliptic lunar orbit is specified by its perilune and apolune altitudes. The hyperbolic transfer trajectory is only partially specified by its total energy E_H and by the constraint that its perilune lies on the earth-moon line.

In this analysis the impulsive braking increment of velocity ΔV_R is applied opposite to the direction of the hyperbolic velocity vector so that the condition is imposed that the hyperbolic and elliptic orbits about the moon be tangent at the braking point. The braking point is defined by $r_R,\theta_{H,R}$ for the hyperbolic orbit and $r_R,\theta_{E,R}$ for the elliptic orbit, where θ is measured clockwise from the perilune of the respective orbits. Since it is desired to examine the effect that the rotation has on the propulsive expense of entry into a specified elliptic orbit the pertinent expressions will be derived in terms of the known elliptic orbit and a hyperbolic orbit of the specified energy that has no rotation associated with it (i.e., the perilune of the hyperbolic orbit is coincident with the perilune of the elliptic orbit). The zero rotation hyperbolic orbital elements are specified by the subscript 0 and may be obtained as follows:

From the condition of coincident perilunes,

$$r_{H,p,0} = r_{E,p}$$

and, from the condition of fixed total energy,

$$V_{H,p,0} = \left[E_H + 2\left(\frac{r_m}{r_{H,p,0}}\right){V_{C,m}}^2\right]^{1/2}$$

where $V_{C,m}$ is the circular satellite velocity at the surface of the moon and is equal to $\left(g_m r_m\right)^{1/2}$; therefore, the eccentricity is

$$\epsilon_{H,0} = \left(\frac{r_{H,p,0}}{r_m}\right)\left(\frac{V_{H,p,0}}{V_{C,m}}\right)^2 - 1$$

and the angle made by the asymptote of the hyperbolic trajectory with the earth-moon line is

$$\beta_{H,0} = \cos^{-1}\left(\frac{1}{\epsilon_{H,0}}\right)$$

The braking velocity increment for zero rotation then is

$$\Delta V_0 = V_{H,p,0} - V_{E,0}$$

where $V_{E,0}$ is the velocity at perilune of the elliptic orbit and may be computed from the expression

$$V_{E,0} = \left[\left(\frac{r_m}{r_{E,p}}\right)\left(1 + \epsilon_E\right)\right]^{1/2} V_{C,m}$$

For the more general tangency condition where the radii and flight-path angles of the hyperbolic and elliptic orbits are equal, the following expressions may be written from the equations for conic sections:

equal radii

$$\frac{r_{H,p,R}\left(\epsilon_{H,R} + 1\right)}{1 + \epsilon_{H,R} \cos \theta_{H,R}} = \frac{r_{E,p}\left(\epsilon_E + 1\right)}{1 + \epsilon_E \cos \theta_{E,R}} \tag{A1}$$

equal flight-path angles

$$\frac{\epsilon_{H,R} \sin \theta_{H,R}}{1 + \epsilon_{H,R} \cos \theta_{H,R}} = \frac{\epsilon_E \sin \theta_{E,R}}{1 + \epsilon_E \cos \theta_{E,R}} \tag{A2}$$

and from figure 22 the angular relationship may be written as

$$\theta_{H,R} - \theta_{E,R} = \theta_R \tag{A3}$$

By use of the fixed hyperbolic energy condition the following expression may be obtained:

$$r_{H,p,R} = \left[\frac{\left(\epsilon_{H,R} - 1\right)}{\left(\epsilon_{H,0} - 1\right)}\right] r_{H,p,0} \tag{A4}$$

Substituting equation (A4) into equation (A1) gives

$$\frac{r_{H,p,0}\left(\epsilon_{H,R} + 1\right)\left(\epsilon_{H,R} - 1\right)}{1 + \epsilon_{H,R} \cos \theta_{H,R}} = \frac{r_{E,p}\left(\epsilon_E + 1\right)\left(\epsilon_{H,0} - 1\right)}{1 + \epsilon_E \cos \theta_{E,R}} \tag{A5}$$

Since $r_{H,p,0} = r_{E,p}$, equation (A5) becomes

$$\frac{\left(\epsilon_{H,R} + 1\right)\left(\epsilon_{H,R} - 1\right)}{1 + \epsilon_{H,R} \cos \theta_{H,R}} = \frac{\left(\epsilon_E + 1\right)\left(\epsilon_{H,0} - 1\right)}{1 + \epsilon_E \cos \theta_{E,R}} \tag{A6}$$

To solve for $\epsilon_{H,R}$ in terms of $\theta_{H,R}$ and $\theta_{E,R}$, first cross-multiply equation (A2) and collect terms so that

$$\epsilon_{H,R}\left[\sin \theta_{H,R} + \epsilon_E\left(\sin \theta_{H,R} \cos \theta_{E,R} - \cos \theta_{H,R} \sin \theta_{E,R}\right)\right] = \epsilon_E \sin \theta_{E,R} \tag{A7}$$

Equation (A7) may be written as

$$\epsilon_{H,R}\left[\sin \theta_{H,R} + \epsilon_E \sin\left(\theta_{H,R} - \theta_{E,R}\right)\right] = \epsilon_E \sin \theta_{E,R} \tag{A8}$$

Substituting θ_R for $\left(\theta_{H,R} - \theta_{E,R}\right)$ in equation (A8) and dividing results in the following expression:

$$\epsilon_{H,R} = \frac{\epsilon_E \sin \theta_{E,R}}{\sin \theta_{H,R} + \epsilon_E \sin \theta_R} \tag{A9}$$

Substituting equation (A9) into equation (A6) gives the following equation:

$$\frac{\left[\epsilon_E \sin\theta_{E,R} + \left(\sin\theta_{H,R} + \epsilon_E \sin\theta_R\right)\right]\left[\epsilon_E \sin\theta_{E,R} - \left(\sin\theta_{H,R} + \epsilon_E \sin\theta_R\right)\right]}{\left[1 + \frac{\epsilon_E \sin\theta_{E,R} \cos\theta_{H,R}}{\left(\sin\theta_{H,R} + \epsilon_E \sin\theta_R\right)}\right]\left(\sin\theta_{H,R} + \epsilon_E \sin\theta_R\right)^2}$$

$$= \frac{\left(\epsilon_E + 1\right)\left(\epsilon_{H,0} - 1\right)}{1 + \epsilon_E \cos\theta_{E,R}} \qquad \text{(A10)}$$

Equation (A10) may be reduced to the following form with the aid of equation (A3):

$$\frac{\epsilon_E^2 \sin^2\theta_{E,R} - \left(\sin\theta_{H,R} + \epsilon_E \sin\theta_R\right)^2}{\left(\sin\theta_{H,R} + \epsilon_E \sin\theta_R\right)\sin\theta_{H,R}} = \left(\epsilon_E + 1\right)\left(\epsilon_{H,0} - 1\right) \qquad \text{(A11)}$$

Now, substituting $\theta_{H,R} - \theta_R$ for $\theta_{E,R}$ in the numerator of equation (A11) and reducing gives

$$\left(A - \cos 2\theta_R\right)\sin\theta_{H,R} + \sin 2\theta_R \cos\theta_{H,R} = -B \sin\theta_R$$

where

$$A = \frac{1 + \left(\epsilon_E + 1\right)\left(\epsilon_{H,0} - 1\right)}{\epsilon_E^2}$$

and

$$B = \frac{2 + \left(\epsilon_E + 1\right)\left(\epsilon_{H,0} - 1\right)}{\epsilon_E}$$

Dividing by $\cos\theta_{H,R}$ and squaring both sides gives

$$\left(A - \cos 2\theta_R\right)^2 \tan^2\theta_{H,R} + 2\left(A - \cos 2\theta_R\right)\sin 2\theta_R \tan\theta_{H,R} + \sin^2 2\theta_R$$

$$= B^2 \sin^2\theta_R\left(1 + \tan^2\theta_{H,R}\right)$$

Collecting terms and solving for $\theta_{H,R}$ results in the expression

$$\theta_{H,R} = \tan^{-1} - \left\{\frac{\left(A - \cos 2\theta_R\right)\sin 2\theta_R + \left[(A - 1)^2 - C^2\sin^2\theta_R\right]^{1/2} B \sin \theta_R}{\left(A - \cos 2\theta_R\right)^2 - B^2\sin^2\theta_R}\right\} \tag{A12}$$

where

$$C = \frac{\left(\epsilon_E + 1\right)\left(\epsilon_{H,O} - 1\right)}{\epsilon_E}$$

It is now possible to completely define the new hyperbolic orbit that will permit the specified rotation of the elliptic orbit. The eccentricity may be determined by the use of equation (A9) which is

$$\epsilon_{H,R} = \frac{\epsilon_E \sin \theta_{E,R}}{\left(\sin \theta_{H,R} + \epsilon_E \sin \theta_R\right)}$$

where

$$\theta_{E,R} = \theta_{H,R} - \theta_R$$

The perilune radius of the new orbit may be obtained from equation (A4) which is

$$r_{H,p,R} = \left[\frac{\left(\epsilon_{H,R} - 1\right)}{\left(\epsilon_{H,O} - 1\right)}\right] r_{H,p,O}$$

The angle made by the asymptote of the new hyperbolic trajectory with the earth-moon line is given by the following equation:

$$\beta_{H,R} = \cos^{-1}\left(\frac{1}{\epsilon_{H,R}}\right) \tag{A13}$$

This completes the definition of the new hyperbolic trajectory.

In order that the hyperbolic and elliptic velocities be determined, the tangency radius r_R may be evaluated as shown in the following equation:

$$r_R = \frac{r_{H,p,R}\left(\epsilon_{H,R} + 1\right)}{1 + \epsilon_{H,R} \cos \theta_{H,R}} \tag{A14}$$

The hyperbolic velocity at the tangency point then is

$$V_{H,R} = \left[2\left(\frac{r_m}{r_R}\right) + \left(\frac{r_m}{r_{H,p,0}}\right)\left(\epsilon_{H,0} - 1\right)\right]^{1/2} V_{C,m} \tag{A15}$$

and the elliptic velocity is

$$V_{E,R} = \left[2\left(\frac{r_m}{r_R}\right) - \left(\frac{r_m}{r_{E,p}}\right)\left(1 - \epsilon_E\right)\right]^{1/2} V_{C,m} \tag{A16}$$

Finally, the impulsive velocity increment required to brake from a hyperbolic orbit of a given energy to a specified elliptic orbit having its major axis at a specified angle θ_R with respect to the earth-moon line is

$$\Delta V_R = V_{H,R} - V_{E,R} \tag{A17}$$

For the case in which 180° rotation of the elliptic orbit is desired, the simpler approach used in computing the zero rotation quantities may be used as shown hereinafter (the subscript π is used to denote the 180° rotation condition). From figure 22 it may be seen that this situation is one in which the perilune of the hyperbolic trajectory is coincident with the apolune of the elliptic orbit

$$r_{H,p,\pi} = r_{E,a}$$

and from the condition of fixed total energy

$$V_{H,p,\pi} = \left[E_H + 2\left(\frac{r_m}{r_{H,p,\pi}}\right)V_{C,m}^2\right]^{1/2}$$

so that the eccentricity is

$$\epsilon_{H,\pi} = \left(\frac{r_{H,p,\pi}}{r_m}\right)\left(\frac{V_{H,p,\pi}}{V_{C,m}}\right)^2 - 1$$

and the angle made by the asymptote of the hyperbolic trajectory with the earth-moon line is

$$\beta_{H,\pi} = \cos^{-1}\left(\frac{1}{\epsilon_{H,\pi}}\right)$$

The braking velocity increment for 180° rotation then is

$$\Delta V_{\pi} = V_{H,p,\pi} - V_{E,\pi}$$

where $V_{E,\pi}$ is the velocity at apolune of the elliptic orbit and may be computed from the expression

$$V_{E,\pi} = \left[\left(\frac{r_m}{r_{E,a}}\right)\left(1 - \epsilon_E\right)\right]^{1/2} V_{C,m}$$

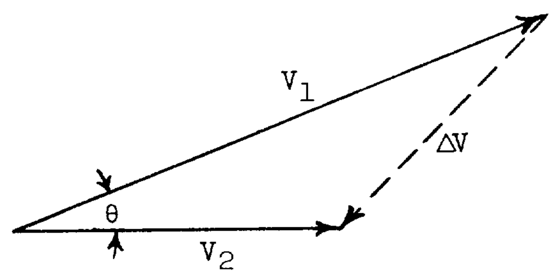
The results of this analysis for the case of an elliptic orbit having a perilune altitude of 50 nautical miles and an apolune altitude of 2,000 nautical miles are presented in figure 23.

APPENDIX B

CONSIDERATION OF A PLANE CHANGE MADE ON ENTRY TO LUNAR ORBIT

Plane changes may be required in order to enter the desired lunar orbit. One way in which such changes may be made without undue cost in fuel expenditure is by appropriate direction of the thrust vector at the time that deceleration is made into lunar orbit. Such a change would be made near perilune of the hyperbolic approach trajectory. Because of this factor such a maneuver may not be desirable for all translunar trajectories.

For the case where perilune of the hyperbolic approach trajectory is near the lunar equator, the trajectory is inclined at an angle θ to the lunar equator, and the desire is to enter lunar orbit in the plane of the lunar equator. The initial velocity V_1 and final velocity V_2 are arranged as shown in the following sketch:



The objective in this appendix is to calculate the difference between the velocity change required to enter an equatorial orbit when θ has a value greater than 0 and when θ has a value equal to 0. From the sketch, this difference is

$$\Delta V_{\theta \neq 0} - \Delta V_{\theta = 0} = \left(V_1^2 + V_2^2 - 2V_1V_2 \cos \theta\right)^{1/2} - \left(V_1 - V_2\right) \tag{B1}$$

This expression may be written in the following form:

$$\Delta V_{\theta \neq 0} - \Delta V_{\theta = 0} = \left(V_1 - V_2\right)\left\{\left[1 + \frac{2V_1V_2(1 - \cos \theta)}{\left(V_1 - V_2\right)^2}\right]^{1/2} - 1\right\}$$

The radical may be expanded in a power series and only the first order terms retained so that

$$\Delta V_{\theta \neq 0} - \Delta V_{\theta = 0} = \frac{V_1 V_2 \theta^2}{2\left(V_1 - V_2\right)} \tag{B2}$$

This formula is restricted by the requirement that V_1 and V_2 be appreciably different and that θ be small.

For $V_1 = 8{,}700$ ft/sec and $V_2 = 5{,}400$ ft/sec, the values in the following table result from the approximate expression (eq. (B2)) and the exact expression (eq. (B1)).

θ, radian	$\Delta V_{\theta \neq 0} - \Delta V_{\theta = 0}$, ft/sec	
	Approximate	Exact
0.05	17.95	17.94
.10	71.18	70.37
.15	160.16	156.16
.25	416.31	444.89
.35	772.66	871.98

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TABLE I

VELOCITY INCREMENTS FOR VARIOUS MISSIONS CONSIDERED

Maximum orbital altitude, h_{max}, nautical miles	Velocity increment, ft/sec, for -								
	Circular lunar orbit			Elliptic lunar orbit, entrance at perilune[a]			Elliptic lunar orbit, entrance at apolune[a]		
	$\Delta V_a, \Delta V_d$	$\Delta V_b, \Delta V_c$	$\Delta V_e, \Delta V_f$	$\Delta V_a, \Delta V_d$	$\Delta V_b, \Delta V_c$	$\Delta V_e, \Delta V_f$	$\Delta V_a, \Delta V_d$	$\Delta V_b, \Delta V_c$	$\Delta V_e, \Delta V_f$
50	3,333	5,649	8,982	3,333	5,649	8,982	3,333	5,649	8,982
100	3,303	5,779	9,083	3,268	5,715	8,982	3,368	5,715	9,083
500	3,145	6,555	9,700	2,857	6,125	8,982	3,579	6,125	9,704
1,000	3,057	7,131	10,187	2,524	6,459	8,982	3,740	6,459	10,198
2,000	3,008	7,728	10,736	2,135	6,847	8,982	3,912	6,847	10,760
4,000	3,041	8,184	11,226	1,772	7,210	8,982	4,056	7,210	11,266
8,000	3,161	8,416	11,577	1,498	7,484	8,982	4,149	7,484	11,633

[a]Perilune distance, 50 nautical miles for elliptic orbits.

TABLE II

PLANE CHANGE AND PILOTING ALLOWANCES IN VELOCITY INCREMENTS

Mission phase	α
Lunar-orbit-rendezvous mission	
Establish and launch from orbit (propulsive efforts a and d)	1.05
Descend and launch to rendezvous (propulsive efforts b and c)	1.25
Direct lunar mission	
Overall allowance (propulsive efforts e and f)	1.15

TABLE III

COMMAND-MODULE WEIGHTS

Mission crew	Weight, lb				
	Fixed	Men and associated equipment	Structural	Heat shield	Total
2	1,000	4,750	1,437	993	8,180
3	1,000	7,125	2,031	1,300	11,456
8	1,000	19,000	5,000	2,500	27,500
14	1,000	33,250	8,563	3,630	46,443

TABLE IV

LUNAR-LANDER-MODULE WEIGHTS

Mission crew	Lander crew	Weight, lb			
		Fixed	Men and associated equipment	Structural	Total
2	1	535	439	244	1,218
3	2	535	878	353	1,766
8	7	535	3,073	902	4,510
14	13	535	5,707	1,561	7,803

TABLE V

SPECIFIC IMPULSES EMPLOYED

Fuel combination	Fuel designation	Braking to orbit	Landing from orbit	Take-off to orbit	Launch from orbit
Lunar-orbit-rendezvous mission					
1	425/425	425	425	425	425
2	425/315	425	315	315	425
3	315/315	315	315	315	315
Direct lunar mission					
1	425/425	425	425	425	425
3	315/315	315	315	315	315

TABLE VI

WEIGHTS OF LUNAR VEHICLES

(a) I = 425 and 425 (see table V)

Type of orbit (a)	Vehicle description (b)	Weight, lb, for -													
		h_{max} = 50 nautical miles		h_{max} = 100 nautical miles		h_{max} = 500 nautical miles		h_{max} = 1,000 nautical miles		h_{max} = 2,000 nautical miles		h_{max} = 4,000 nautical miles		h_{max} = 8,000 nautical miles	
		W_S = 0 lb	W_S = 40,000 lb	W_S = 0 lb	W_S = 40,000 lb	W_S = 0 lb	W_S = 40,000 lb	W_S = 0 lb	W_S = 40,000 lb	W_S = 0 lb	W_S = 40,000 lb	W_S = 0 lb	W_S = 40,000 lb	W_S = 0 lb	W_S = 40,000 lb
Two-man crew															
A	LORV	26,793	198,018	26,965	200,644	28,320	218,081	29,783	233,498	31,881	252,628	34,159	270,919	35,997	284,002
	LLV	5,909	120,502	6,114	122,695	7,509	136,879	8,787	148,881	10,391	162,923	11,851	174,920	12,687	181,507
	DLV	81,493	246,497	83,738	251,037	99,264	281,689	113,997	309,749	133,887	346,382	155,287	384,514	173,312	415,793
C	LORV	26,793	198,018	26,654	198,250	25,894	199,957	25,413	201,672	25,002	204,061	24,766	206,684	24,684	208,931
	LLV	5,909	120,502	6,011	121,600	6,695	128,763	7,318	135,002	8,130	142,805	8,983	150,649	9,698	156,974
	DLV	81,493	246,497	81,493	246,497	81,493	246,497	81,493	246,497	81,493	246,497	81,493	246,497	81,493	246,497
B	LORV	26,793	198,018	27,102	200,410	29,116	215,993	30,866	229,489	33,037	246,177	35,184	262,624	36,875	275,553
	LLV	5,909	20,502	6,011	121,600	6,695	128,763	7,318	135,002	8,130	142,805	8,983	150,649	9,698	156,974
	DLV	81,493	246,497	83,741	251,045	99,372	281,898	114,352	310,416	134,829	348,087	157,219	387,900	176,397	421,080
Three-man crew															
A	LORV	37,790	209,015	38,045	211,724	40,042	229,802	42,180	245,895	45,232	265,979	48,532	285,292	51,176	299,181
	LLV	8,573	123,166	8,870	125,451	10,894	140,264	12,748	152,842	15,075	167,607	17,193	180,261	18,406	187,226
	DLV	114,139	279,143	117,283	284,582	139,029	321,453	159,663	355,415	187,521	400,016	217,494	446,720	242,739	485,221
C	LORV	37,790	209,015	37,601	209,197	36,575	210,638	35,934	212,194	35,400	214,460	35,112	217,030	35,030	219,278
	LLV	8,573	123,166	8,721	124,310	9,713	131,781	10,616	138,300	11,794	146,470	13,032	154,698	14,069	161,345
	DLV	114,139	279,143	114,139	279,143	114,139	279,143	114,139	279,143	114,139	279,143	114,139	279,143	114,139	279,143
B	LORV	37,790	209,015	38,231	211,539	41,110	227,987	43,614	242,238	46,727	259,866	49,809	277,249	52,240	290,918
	LLV	8,573	123,166	8,721	124,310	9,713	131,781	10,616	138,300	11,794	146,470	13,032	154,698	14,069	161,345
	DLV	114,139	279,143	117,287	284,590	139,180	321,706	160,161	356,224	188,841	402,099	220,200	450,881	247,061	491,744
Eight-man crew															
A	LORV	92,016	263,241	92,691	266,370	97,918	287,679	103,448	307,163	111,279	332,026	119,680	356,439	126,337	374,342
	LLV	21,892	136,485	22,649	139,230	27,819	157,188	32,552	172,646	38,493	191,025	43,902	206,970	47,000	215,819
	DLV	273,983	438,987	281,531	448,830	333,730	516,154	385,260	579,012	450,130	662,625	522,079	751,306	582,679	825,160
C	LORV	92,016	263,241	91,588	263,184	89,289	263,351	87,892	264,152	86,792	265,851	86,281	268,198	86,230	270,478
	LLV	21,892	136,485	22,269	137,858	24,803	146,871	27,108	154,792	30,116	164,791	33,277	174,943	35,924	183,200
	DLV	273,983	438,987	273,983	438,987	273,983	438,987	273,983	438,987	273,983	438,987	273,983	438,987	273,983	438,987
B	LORV	92,016	263,241	93,112	266,421	100,283	287,161	106,536	305,160	114,324	327,464	122,058	349,498	128,175	366,853
	LLV	21,892	136,485	22,269	137,858	24,803	146,871	27,108	154,792	30,116	164,791	33,277	174,943	35,924	183,200
	DLV	273,983	438,987	281,541	448,843	334,093	516,618	384,456	580,519	453,300	666,557	528,574	759,256	593,052	837,735
Fourteen-man crew															
A	LORV	156,397	327,621	157,579	331,259	166,704	356,465	176,315	380,030	189,687	410,634	204,404	441,164	215,861	463,866
	LLV	37,874	152,467	39,184	155,766	48,128	177,497	56,317	196,411	66,595	219,128	75,953	239,021	81,312	250,131
	DLV	462,714	627,717	475,460	642,759	563,615	746,040	647,264	843,016	760,197	972,692	881,707	1,110,933	984,050	1,226,531
C	LORV	156,397	327,621	155,690	327,286	151,917	325,979	149,653	325,913	147,914	326,973	147,172	329,090	147,186	331,434
	LLV	37,874	152,467	38,527	154,116	42,911	164,978	46,898	174,583	52,102	186,778	57,570	199,237	62,150	209,427
	DLV	462,714	627,717	462,714	627,717	462,714	627,717	462,714	627,717	462,714	627,717	462,714	627,717	462,714	627,717
B	LORV	156,397	327,621	158,275	331,583	170,566	357,444	181,294	379,918	194,666	407,806	207,960	435,400	218,485	457,163
	LLV	37,874	152,467	38,527	154,116	42,911	164,978	46,898	174,583	52,102	186,778	57,570	199,237	62,150	209,427
	DLV	462,714	627,717	475,477	642,779	564,229	746,754	649,283	845,347	765,549	978,807	892,676	1,123,358	1,001,569	1,246,252

[a]A refers to circular orbit with altitude equal to h_{max}, B refers to elliptic orbit entered at apolune altitude equal to h_{max} (perilune altitude equal to 50 nautical miles), and C refers to elliptic orbit entered at perilune altitude equal to 50 nautical miles (apolune altitude equal to h_{max}).

[b]LORV refers to lunar-orbital-rendezvous vehicle, LLV refers to lunar-lander vehicle, and DLV refers to direct lunar vehicle.

TABLE VI.- Continued

WEIGHTS OF LUNAR VEHICLES

(b) I = 425 and 315 (see table V)

Type of orbit (a)	Vehicle description (b)	Weight, lb, for - h_{max} = 50 nautical miles, W_S = 0 lb	h_{max} = 50 nautical miles, W_S = 40,000 lb	h_{max} = 100 nautical miles, W_S = 0 lb	h_{max} = 100 nautical miles, W_S = 40,000 lb	h_{max} = 500 nautical miles, W_S = 0 lb	h_{max} = 500 nautical miles, W_S = 40,000 lb	h_{max} = 1,000 nautical miles, W_S = 0 lb	h_{max} = 1,000 nautical miles, W_S = 40,000 lb	h_{max} = 2,000 nautical miles, W_S = 0 lb	h_{max} = 2,000 nautical miles, W_S = 40,000 lb	h_{max} = 4,000 nautical miles, W_S = 0 lb	h_{max} = 4,000 nautical miles, W_S = 40,000 lb	h_{max} = 8,000 nautical miles, W_S = 0 lb	h_{max} = 8,000 nautical miles, W_S = 40,000 lb
							Two-man crew								
A	LORV	33,027	257,463	33,647	263,189	38,454	302,728	43,688	339,938	51,426	388,872	59,885	437,464	65,984	470,795
	LLV	10,081	160,286	10,600	164,678	14,418	194,587	18,350	222,080	23,897	257,066	29,570	289,628	33,099	308,658
	DLV	81,493	246,497	83,738	251,037	99,264	281,689	113,997	309,749	133,887	346,382	155,287	384,514	173,312	415,793
C	LORV	33,027	257,463	33,078	258,934	33,646	268,951	34,449	278,249	35,830	290,606	37,616	303,844	39,339	315,116
	LLV	10,081	160,286	10,338	162,477	12,131	177,147	13,864	190,475	16,273	207,900	18,989	226,312	21,412	241,852
	DLV	81,493	246,497	81,493	246,497	81,493	246,497	81,493	246,497	81,493	246,497	81,493	246,497	81,493	246,497
B	LORV	33,027	257,463	33,589	261,699	37,438	290,068	41,049	315,781	45,925	349,196	51,248	384,097	55,860	413,106
	LLV	10,081	160,286	10,338	162,477	12,131	177,147	13,864	190,475	16,273	207,900	18,989	226,312	21,412	241,852
	DLV	81,493	246,497	83,741	251,043	99,372	281,898	114,352	310,416	134,829	348,087	157,219	387,900	176,397	421,080
							Three-man crew								
A	LORV	46,833	271,269	47,739	277,281	54,743	319,017	62,353	358,603	73,587	411,033	85,853	463,432	94,678	499,489
	LLV	14,625	164,830	15,377	169,456	20,917	201,086	26,621	230,351	34,668	267,837	42,898	302,956	48,018	323,577
	DLV	114,139	279,143	117,283	284,582	139,029	321,453	159,663	355,415	187,521	400,016	217,494	446,720	242,739	485,221
C	LORV	46,833	271,269	46,920	272,777	47,820	283,126	49,044	292,843	51,108	305,887	53,753	319,981	56,291	332,068
	LLV	14,625	164,830	14,998	167,137	17,599	182,616	20,113	196,724	23,608	215,234	27,548	234,871	31,064	251,503
	DLV	114,139	279,143	114,139	279,143	114,139	279,143	114,139	279,143	114,139	279,143	114,139	279,143	114,139	279,143
B	LORV	46,833	271,269	47,643	275,753	53,183	305,812	58,388	333,120	65,424	368,694	73,114	405,963	79,782	437,029
	LLV	14,625	164,830	14,998	167,137	17,599	182,616	20,113	196,724	23,608	215,234	27,548	234,871	31,064	251,503
	DLV	114,139	279,143	117,287	284,590	139,180	321,706	160,161	356,224	188,841	402,099	220,200	450,881	247,061	491,744
							Eight-man crew								
A	LORV	115,106	339,543	117,445	346,987	135,457	399,731	154,957	451,207	183,682	521,128	214,975	592,554	237,415	642,226
	LLV	37,345	187,550	39,265	193,344	53,411	233,580	67,975	271,705	88,523	321,692	109,537	369,595	122,612	398,170
	DLV	273,983	438,987	281,531	448,830	333,730	516,154	383,260	579,012	450,130	662,625	522,079	751,306	582,679	825,160
C	LORV	115,106	339,543	115,383	341,239	118,003	353,308	121,368	365,167	126,900	381,678	133,879	400,107	140,519	416,296
	LLV	37,345	187,550	38,298	190,437	44,940	209,956	51,358	227,968	60,282	251,909	70,343	277,666	79,319	299,759
	DLV	273,983	438,987	273,983	438,987	273,983	438,987	273,983	438,987	273,983	438,987	273,983	438,987	273,983	438,987
B	LORV	115,106	339,543	117,145	345,255	131,111	383,741	144,259	418,991	162,066	465,337	161,567	514,415	198,503	555,749
	LLV	37,345	187,550	38,298	190,437	44,940	209,956	51,358	227,968	60,282	251,909	70,343	277,666	79,319	299,759
	DLV	273,983	438,987	281,541	448,843	334,093	516,618	384,456	580,519	453,300	666,557	528,574	759,256	593,052	837,735
							Fourteen-man crew								
A	LORV	196,344	420,780	200,406	429,948	231,648	495,921	265,428	561,678	315,148	625,594	369,269	746,849	408,032	812,842
	LLV	64,609	214,814	67,931	222,010	92,403	272,572	117,600	321,330	153,148	386,317	189,504	449,562	212,124	487,663
	DLV	462,714	627,717	475,460	642,759	563,615	746,040	647,264	843,016	760,197	972,692	881,707	1,110,933	964,050	1,226,551
C	LORV	196,344	420,780	196,856	422,713	201,592	436,898	207,567	451,366	217,303	472,081	229,519	495,748	241,109	516,88[illegible]
	LLV	64,609	214,814	66,257	218,396	77,748	242,764	88,852	265,462	104,291	295,918	121,697	329,020	137,227	357,666
	DLV	462,714	627,717	462,714	627,717	462,714	627,717	462,714	627,717	462,714	627,717	462,714	627,717	462,714	627,717
B	LORV	196,344	420,780	199,852	427,962	223,899	476,529	246,556	521,289	277,262	580,533	310,913	643,761	340,154	697,400
	LLV	64,609	214,814	66,257	218,396	77,748	242,764	88,852	265,462	104,291	295,918	121,697	329,020	137,227	357,666
	DLV	462,714	627,717	475,477	642,779	564,229	746,754	649,283	845,347	765,549	978,807	892,676	1,123,358	1,001,569	1,246,252

[a]A refers to circular orbit with altitude equal to h_{max}, B refers to elliptic orbit entered at apolune altitude equal to h_{max} (perilune altitude equal to 50 nautical miles), and C refers to elliptic orbit entered at perilune altitude equal to 50 nautical miles (apolune altitude equal to h_{max}).

[b]LORV refers to lunar-orbital-rendezvous vehicle, LLV refers to lunar-lander vehicle, and DLV refers to direct lunar vehicle.

TABLE VI.- Concluded

WEIGHTS OF LUNAR VEHICLES

(c) I = 315 and 315 (see table V)

Type of orbit (a)	Vehicle description (b)	Weight, lb, for - h_{max} = 50 nautical miles, W_S = 0 lb	h_{max} = 50 nautical miles, W_S = 40,000 lb	h_{max} = 100 nautical miles, W_S = 0 lb	h_{max} = 100 nautical miles, W_S = 40,000 lb	h_{max} = 500 nautical miles, W_S = 0 lb	h_{max} = 500 nautical miles, W_S = 40,000 lb	h_{max} = 1,000 nautical miles, W_S = 0 lb	h_{max} = 1,000 nautical miles, W_S = 40,000 lb	h_{max} = 2,000 nautical miles, W_S = 0 lb	h_{max} = 2,000 nautical miles, W_S = 40,000 lb	h_{max} = 4,000 nautical miles, W_S = 0 lb	h_{max} = 4,000 nautical miles, W_S = 40,000 lb	h_{max} = 8,000 nautical miles, W_S = 0 lb	h_{max} = 8,000 nautical miles, W_S = 40,000 lb
Two-man crew															
A	LORV	39,633	291,763	40,243	297,814	45,158	339,936	50,717	380,066	59,153	433,618	68,665	488,183	75,930	527,730
	LLV	10,081	160,286	10,600	164,678	14,418	194,587	18,350	222,080	23,897	257,066	29,570	289,628	33,099	308,658
	DLV	206,399	471,605	215,892	487,294	287,958	602,726	367,485	724,526	494,302	910,749	659,604	1,143,756	826,086	1,370,983
C	LORV	39,633	291,763	39,502	292,594	39,026	298,681	39,083	304,841	39,680	313,552	40,799	323,345	42,045	331,957
	LLV	10,081	160,286	10,338	162,477	12,131	177,147	13,864	190,475	16,273	207,900	18,989	226,312	21,412	241,852
	DLV	206,399	471,605	206,399	471,605	206,399	471,605	206,399	471,605	206,399	471,605	206,399	471,605	206,399	471,605
B	LORV	39,633	291,763	40,368	296,966	45,370	331,860	50,031	363,540	56,270	404,705	63,004	447,605	68,770	483,113
	LLV	10,081	160,286	10,338	162,477	12,131	177,147	13,864	190,475	16,273	207,900	18,989	226,312	21,412	241,852
	DLV	206,399	471,605	215,905	487,315	288,500	603,574	369,548	727,626	500,918	920,259	676,179	116,666	85,739	1,413,076
Three-man crew															
A	LORV	56,157	308,287	57,052	314,623	64,235	359,012	72,328	401,677	84,581	459,046	98,369	517,887	108,871	560,671
	LLV	14,625	164,830	15,377	169,456	20,917	201,086	26,621	230,351	34,668	267,837	42,898	302,956	48,018	323,577
	DLV	289,080	554,287	302,377	573,779	403,311	718,079	514,696	871,737	692,315	1,108,762	923,835	1,407,987	1,157,009	1,701,906
C	LORV	56,157	308,287	55,990	309,081	55,429	315,083	55,605	321,363	56,570	330,442	58,277	340,823	60,142	350,054
	LLV	14,625	164,830	14,998	167,137	17,599	182,616	20,113	196,724	23,608	215,234	27,548	234,871	31,064	251,503
	DLV	289,080	554,287	289,080	554,287	289,080	554,287	289,080	554,287	289,080	554,287	289,080	554,287	289,080	554,287
B	LORV	56,157	308,287	57,211	313,809	64,392	350,883	71,094	384,603	80,077	428,511	89,787	474,388	98,111	512,454
	LLV	14,625	164,830	14,998	167,137	17,599	182,616	20,113	196,724	23,608	215,234	27,548	234,871	31,064	251,503
	DLV	289,080	554,287	302,395	573,805	404,070	719,144	517,586	875,664	701,582	1,120,923	947,050	1,437,538	1,200,859	1,756,540
Eight-man crew															
A	LORV	137,820	389,950	140,149	397,719	158,709	453,487	179,492	508,841	210,845	585,309	246,007	665,526	272,662	724,463
	LLV	37,345	187,550	39,265	193,344	53,411	233,580	67,975	271,705	88,523	321,692	109,537	369,595	122,612	398,170
	DLV	693,917	959,123	725,835	997,237	968,119	1,282,888	1,235,490	1,592,531	1,661,852	2,078,300	2,217,599	2,701,752	2,777,317	3,322,214
C	LORV	137,820	389,950	137,486	390,577	136,598	396,252	137,444	403,203	140,326	414,199	145,034	427,581	150,041	439,952
	LLV	37,345	187,550	38,298	190,437	44,940	209,956	51,358	227,968	60,282	251,909	70,343	277,666	79,319	299,759
	DLV	693,917	959,123	693,917	959,123	693,917	959,123	693,917	959,123	693,917	959,123	693,917	959,123	693,917	959,123
B	LORV	137,820	389,950	140,460	397,057	158,481	444,972	175,339	488,848	197,987	546,422	222,532	607,132	243,616	657,959
	LLV	37,345	187,550	38,298	190,437	44,940	209,956	51,358	227,968	60,282	251,909	70,343	277,666	79,319	299,759
	DLV	693,917	959,123	725,878	997,288	969,942	1,285,016	1,242,429	1,600,507	1,684,097	2,103,438	2,273,325	2,763,814	2,882,575	3,438,256
Fourteen-man crew															
A	LORV	234,944	487,074	238,999	496,570	271,250	566,027	307,280	636,629	361,562	736,027	422,368	841,886	468,380	920,180
	LLV	64,609	214,814	67,931	222,010	92,403	272,572	117,600	321,330	153,148	386,317	189,504	449,562	212,124	487,683
	DLV	1,171,912	1,437,119	1,225,817	1,497,219	1,634,996	1,949,764	2,086,542	2,443,583	2,806,597	3,223,045	3,745,164	4,229,316	4,690,436	5,235,333
C	LORV	234,944	487,074	234,425	487,516	233,236	492,890	234,952	500,710	240,202	514,075	248,569	531,116	257,383	547,295
	LLV	64,609	214,814	66,257	218,396	77,748	242,764	88,852	265,462	104,291	295,918	121,697	329,020	137,227	357,666
	DLV	1,171,912	1,437,119	1,171,912	1,437,119	1,171,912	1,437,119	1,171,912	1,437,119	1,171,912	1,437,119	1,171,912	1,437,119	1,171,912	1,437,119
B	LORV	234,944	487,074	239,478	496,076	270,455	556,946	299,458	612,967	338,457	686,891	380,760	765,361	417,129	831,472
	LLV	64,609	214,814	66,257	218,396	77,748	242,764	88,852	265,462	104,291	295,918	121,697	329,020	137,227	357,666
	DLV	1,171,912	1,437,119	1,225,889	1,497,299	1,638,074	1,953,148	2,098,260	2,456,338	2,844,166	3,263,506	3,839,274	4,329,763	4,868,199	5,423,880

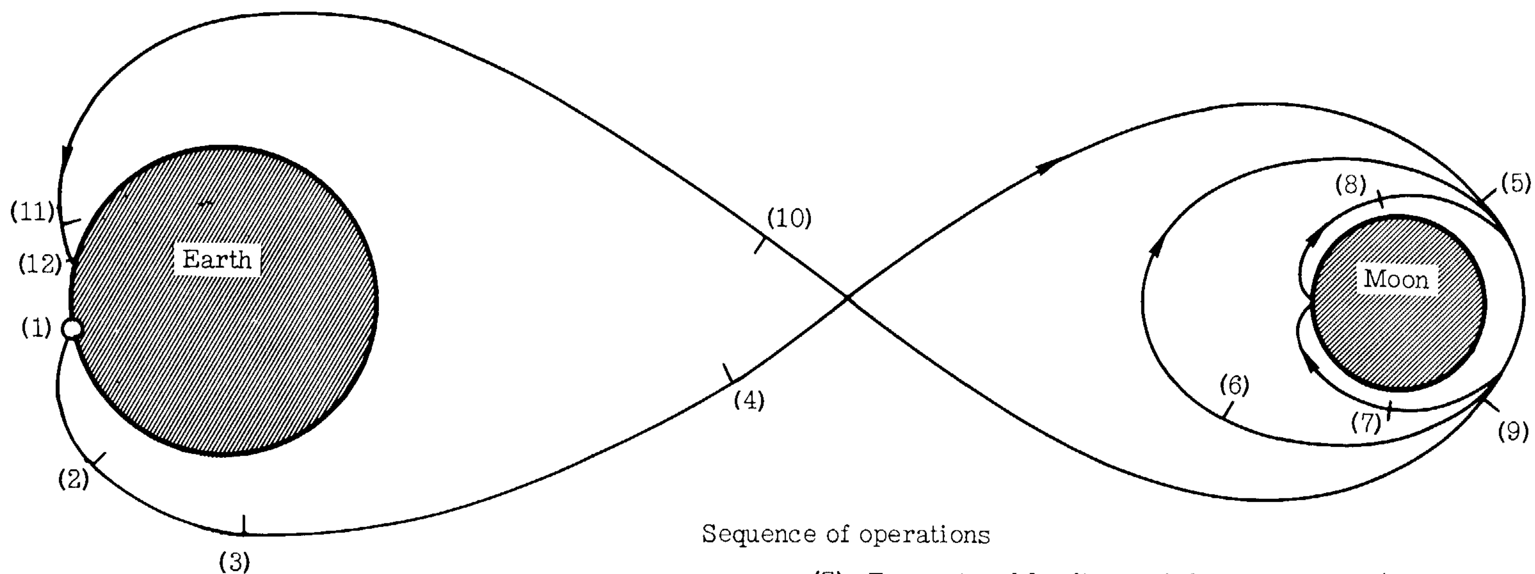
[a] A refers to circular orbit with altitude equal to h_{max}, B refers to elliptic orbit entered at apolune altitude equal to h_{max} (perilune altitude equal to 50 nautical miles), and C refers to elliptic orbit entered at perilune altitude equal to 50 nautical miles (apolune altitude equal to h_{max}).

[b] LORV refers to lunar-orbital-rendezvous vehicle, LLV refers to lunar-lander vehicle, and DLV refers to direct lunar vehicle.

TABLE VII

COMPARISON OF INITIAL WEIGHTS OF ONE-STAGE AND TWO-STAGE LUNAR LANDERS FOR A THREE-MAN MISSION USING A 100-NAUTICAL-MILE CIRCULAR LUNAR ORBIT

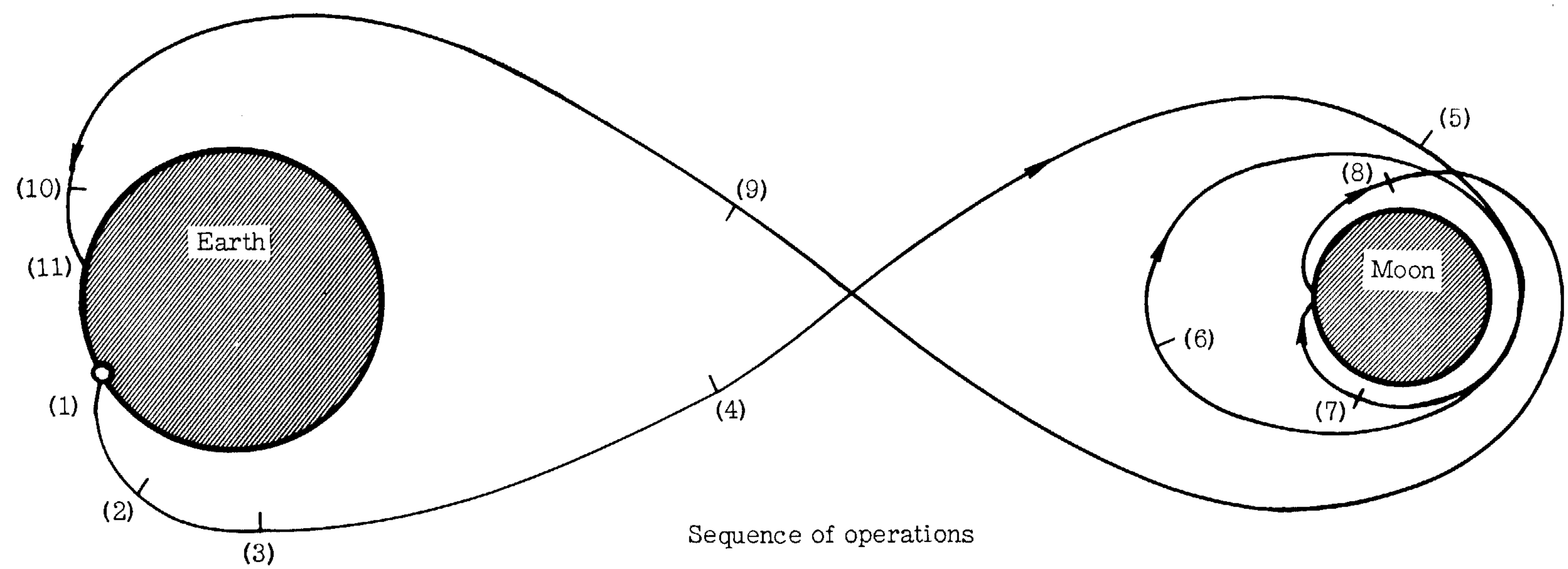
Vehicle	Weight, lb, for -	
	W_S = 0 lb	W_S = 40,000 lb
I = 425 seconds		
One-stage lander	11,032	181,012
Two-stage lander	8,870	125,451
I = 315 seconds		
One-stage lander	37,691	500,735
Two-stage lander	15,377	169,456



Sequence of operations

(1) Launch from earth
(2) Circularization of orbit and establishment of ephemeris
(3) Injection to moon
(4) Midcourse correction
(5) Establishment of lunar orbit (impulsive effort a)
(6) Coast in lunar orbit and establishment of ephemeris
(7) Descent and landing with lander vehicle (impulsive effort b)
(8) Take-off and rendezvous with orbiting command module (impulsive effort c)
(9) Launch to earth return in command module (impulsive effort d)
(10) Midcourse correction
(11) Reentry
(12) Landing on earth

Figure 1.- Mission profile for lunar-orbit-rendezvous mission.



Sequence of operations

(1) Launch from earth
(2) Circularization of orbit and establishment of ephemeris
(3) Injection to moon
(4) Midcourse correction
(5) Establishment of lunar orbit (included in impulsive effort e)
(6) Coast in lunar orbit and establishment of ephemeris
(7) Descent and landing with entire vehicle (included in impulsive effort e)
(8) Take-off and launch to earth return (impulsive effort f)
(9) Midcourse correction
(10) Reentry
(11) Landing on earth

Figure 2.- Mission profile for direct lunar mission.

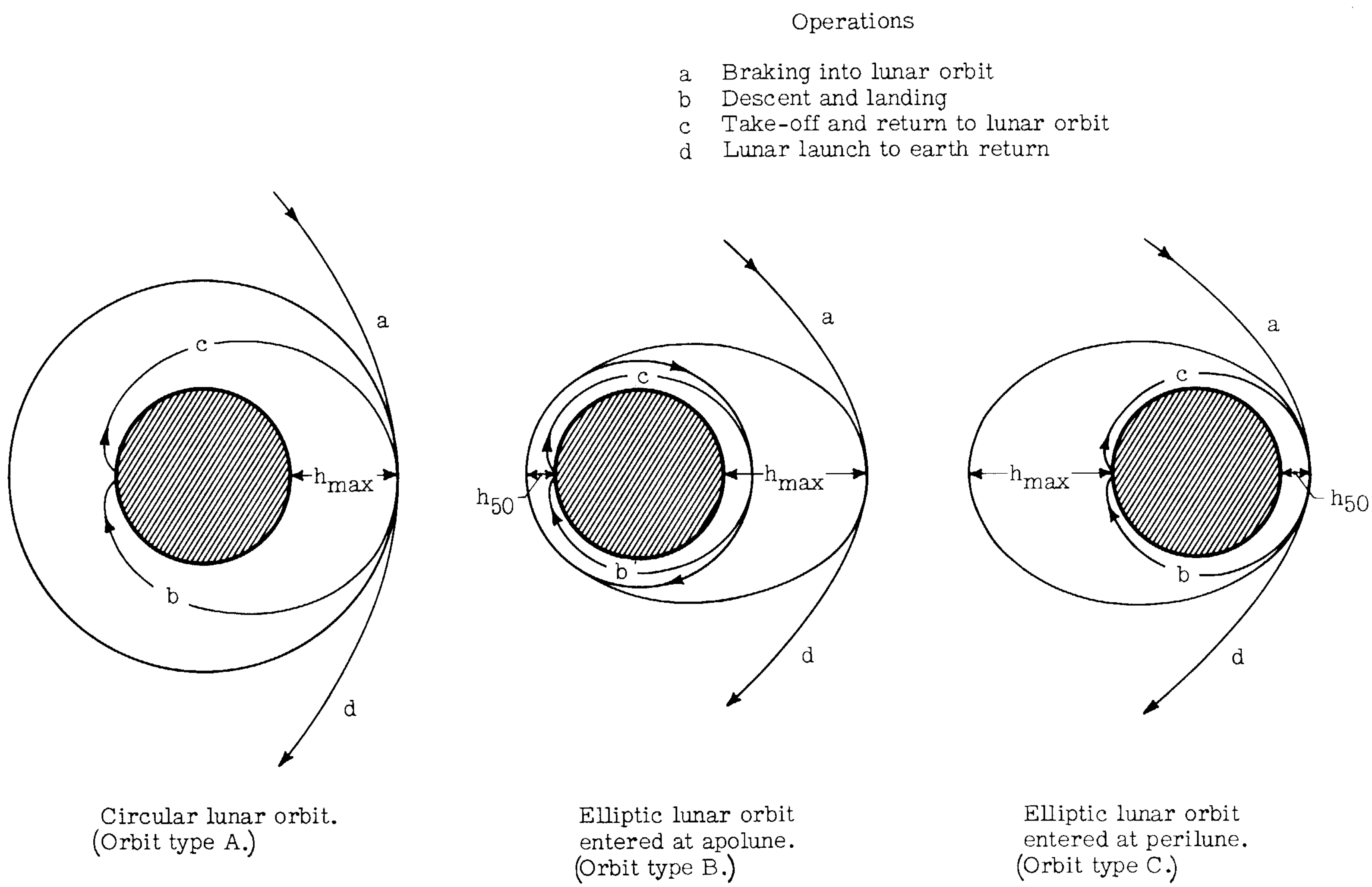
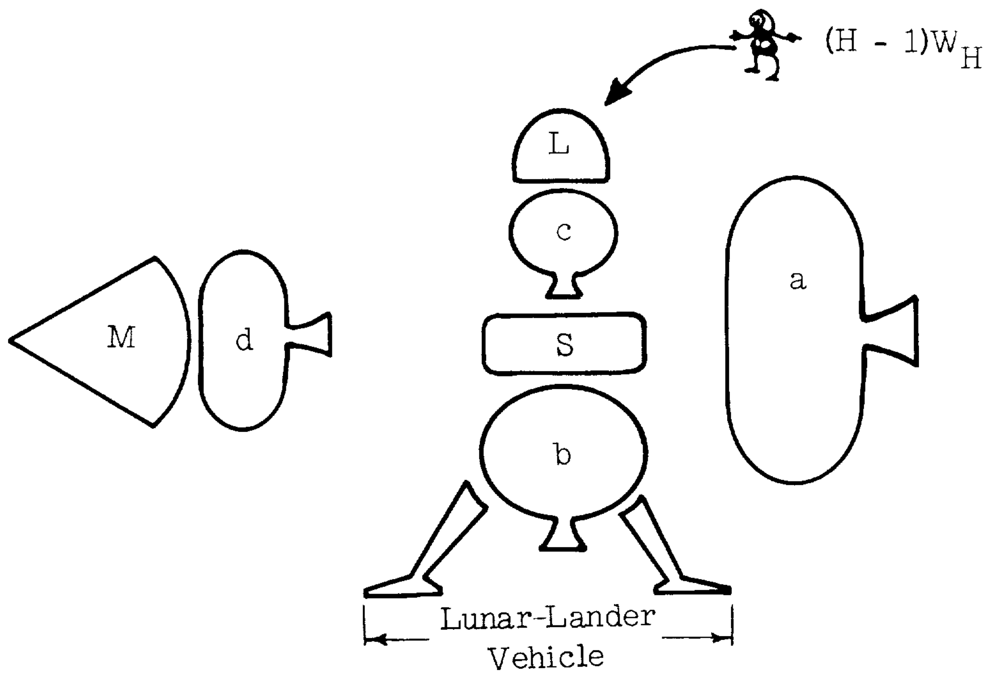
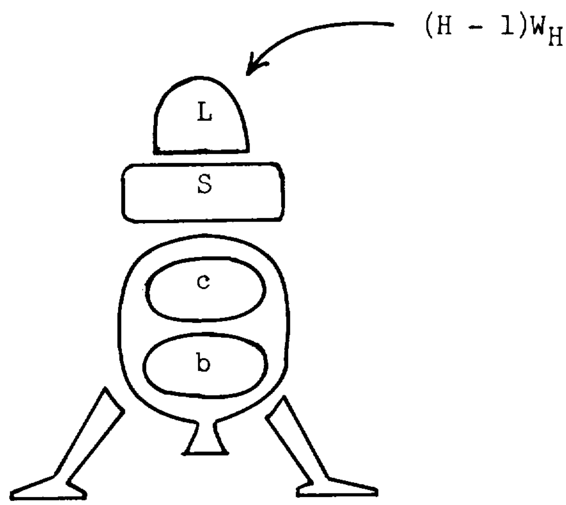


Figure 3.- Types of lunar orbits considered in investigation.



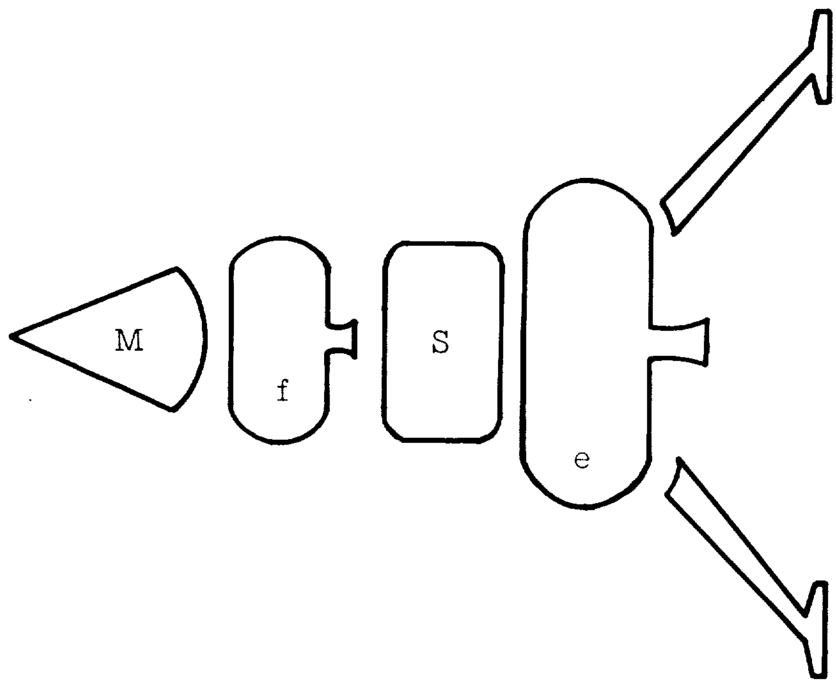
Propulsive element	k_G	k_C	k_T
a	0	.080	.111
b	.060	.080	.111
c	0	.080	.111
d	0	.080	.111

Figure 4.- Schematic of lunar-orbit-rendezvous vehicle. $k_S = 0.250$, $W_H = 200$ pounds.



k_G	k_C	k_T	k_S
.06	.08	.111	.250

Figure 5.- Schematic of single-stage lunar lander. W_H = 200 pounds.



Propulsive element	k_G	k_C	k_T
e	0.060	.080	.111
f	0	.080	.111

Figure 6.- Schematic of direct lunar-mission vehicle. k_S = 0.250.

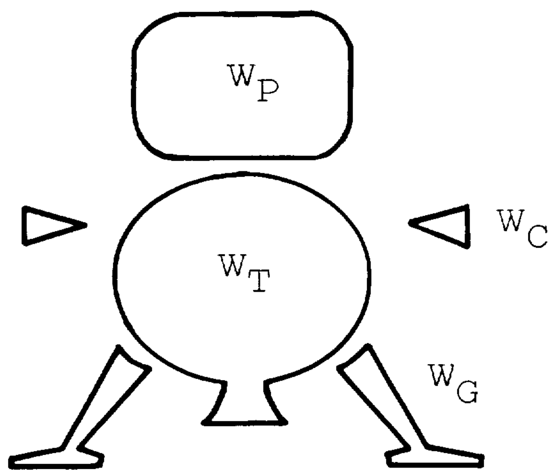


Figure 7.- Schematic of unit rocket system.

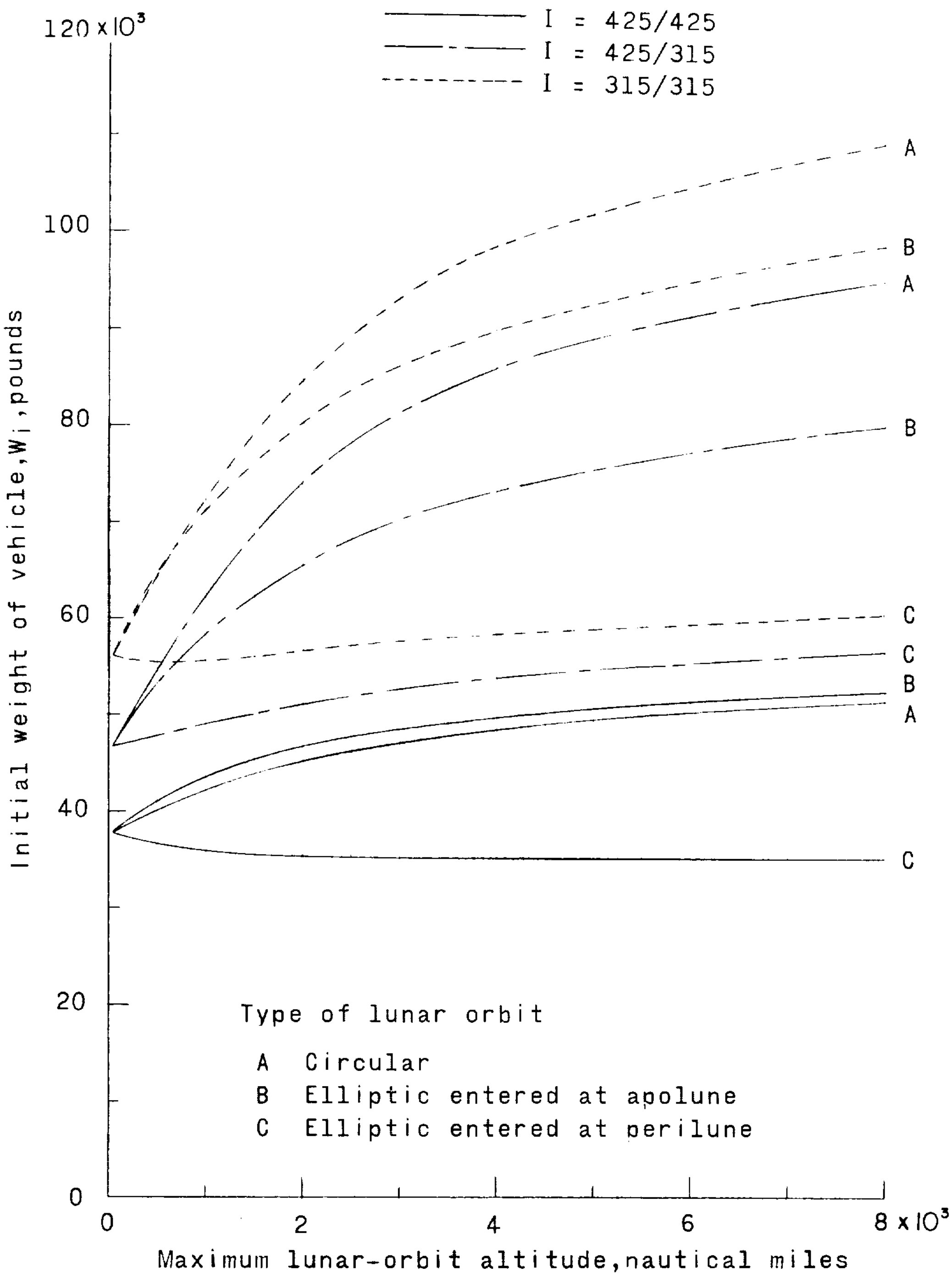


Figure 8.- Weight of lunar-orbit-rendezvous vehicle in transit to moon as a function of maximum lunar-orbit altitude for three types of lunar orbit and three combinations of fuel. Three-man crew; transported weight, 0 pound.

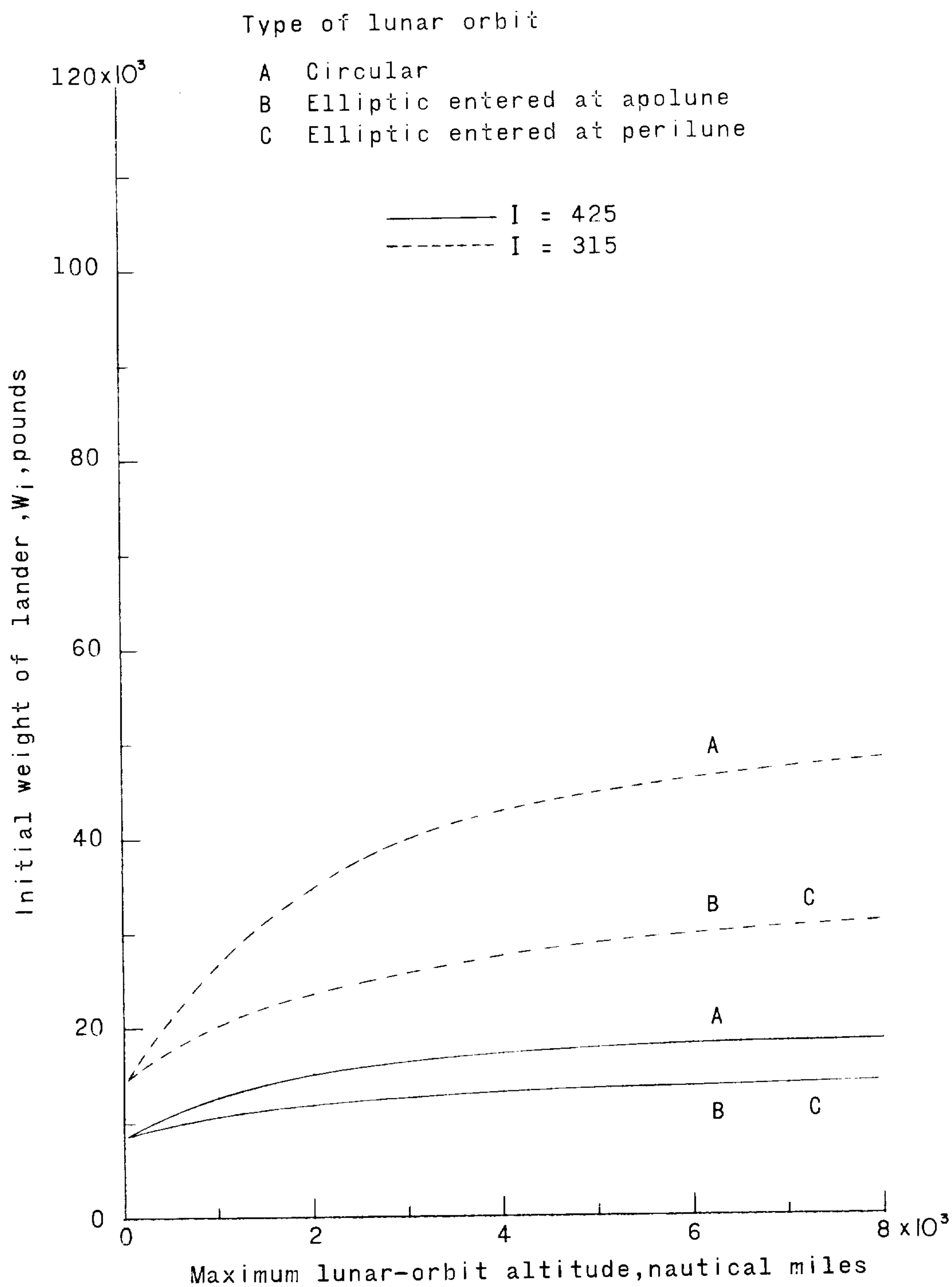


Figure 9.- Weight of lunar lander prior to descent to lunar surface as a function of maximum lunar-orbit altitude for three types of lunar orbit and two different fuels. Three-man crew; transported weight, 0 pound.

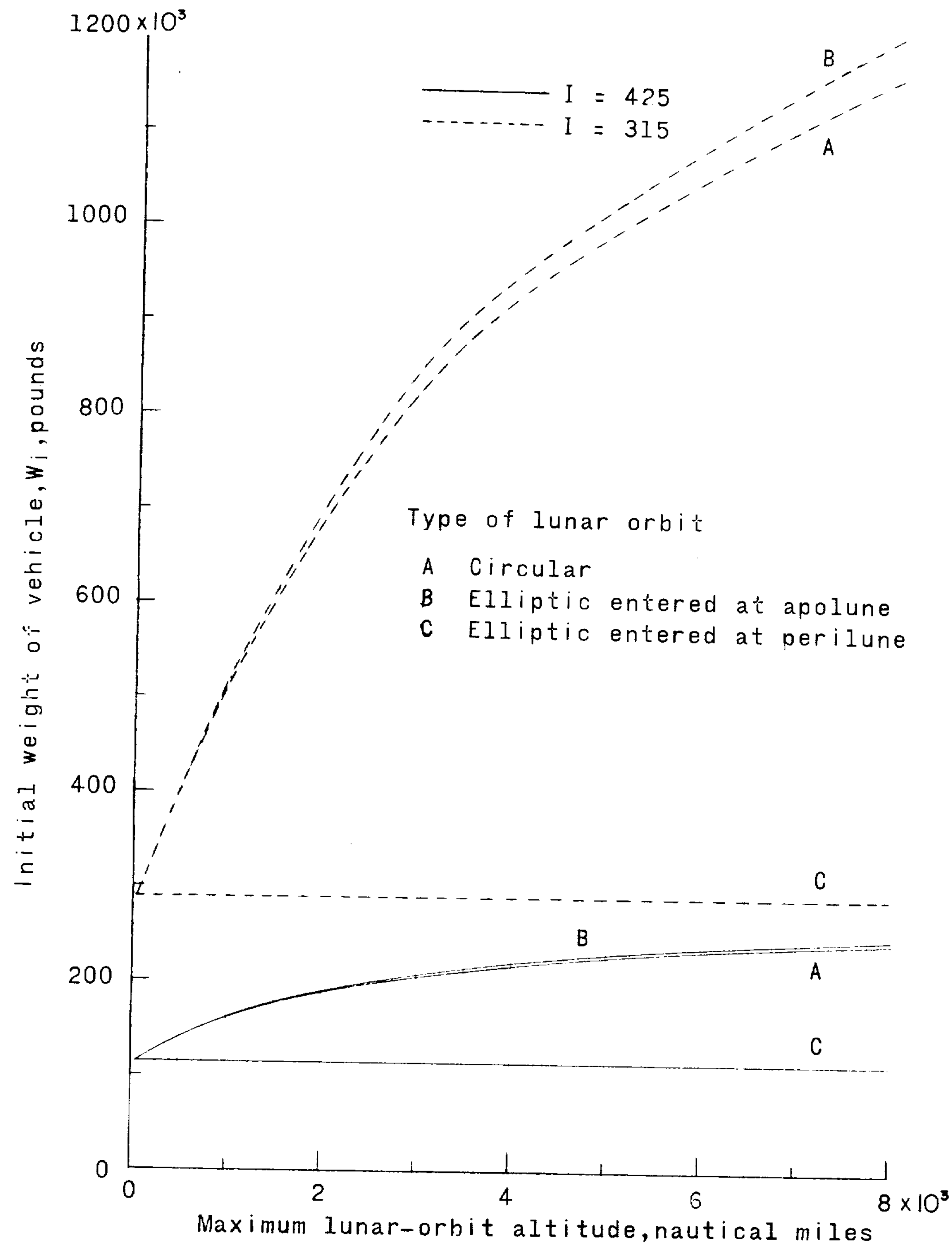


Figure 10.- Weight of direct lunar vehicle in transit to moon as a function of maximum lunar-orbit altitude for three types of lunar orbit and two different fuels. Three-man crew; transported weight, 0 pound.

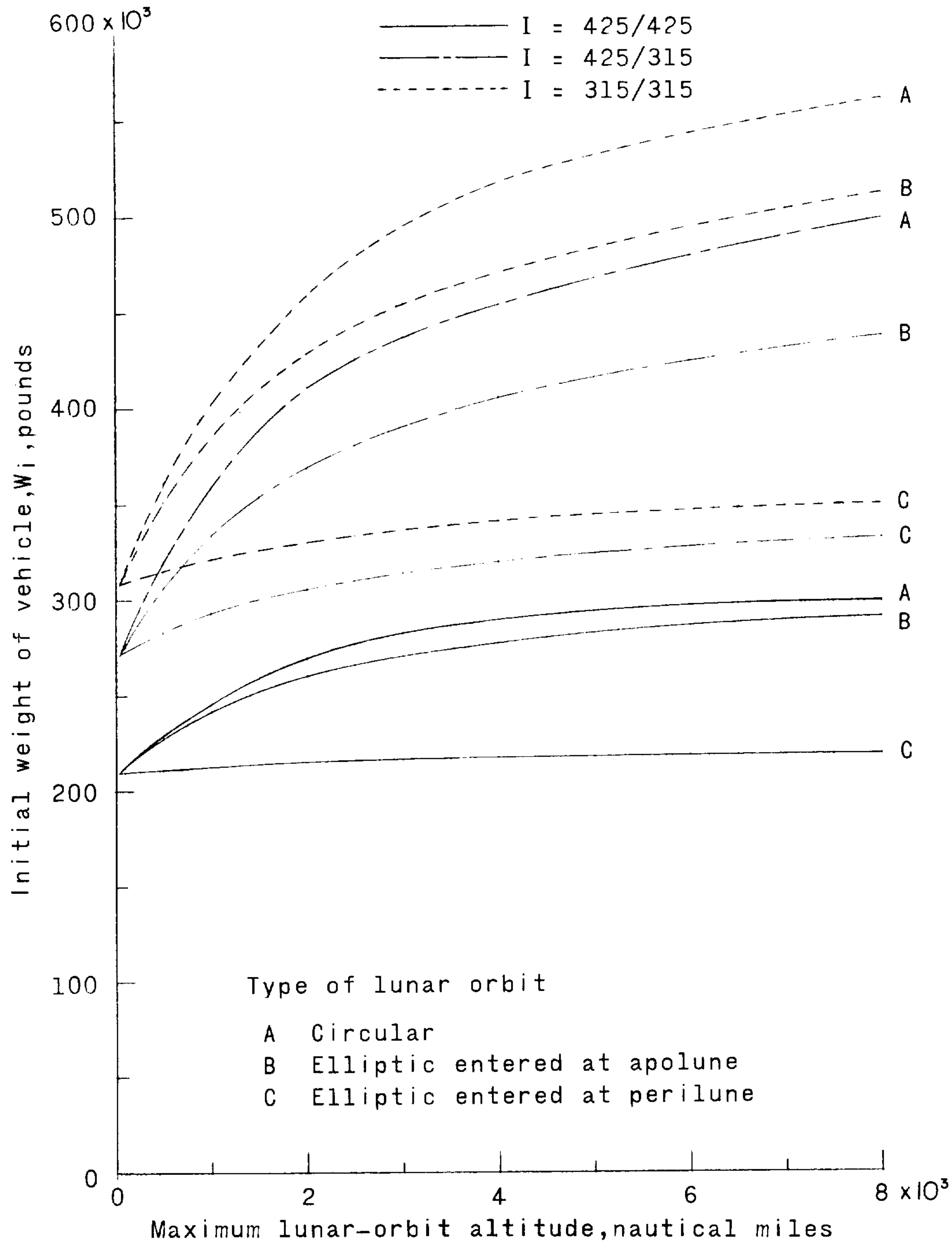


Figure 11.- Weight of lunar-orbit-rendezvous vehicle in transit to moon as a function of maximum lunar-orbit altitude for three types of lunar orbit and three combinations of fuel. Three-man crew; transported weight, 40,000 pounds.

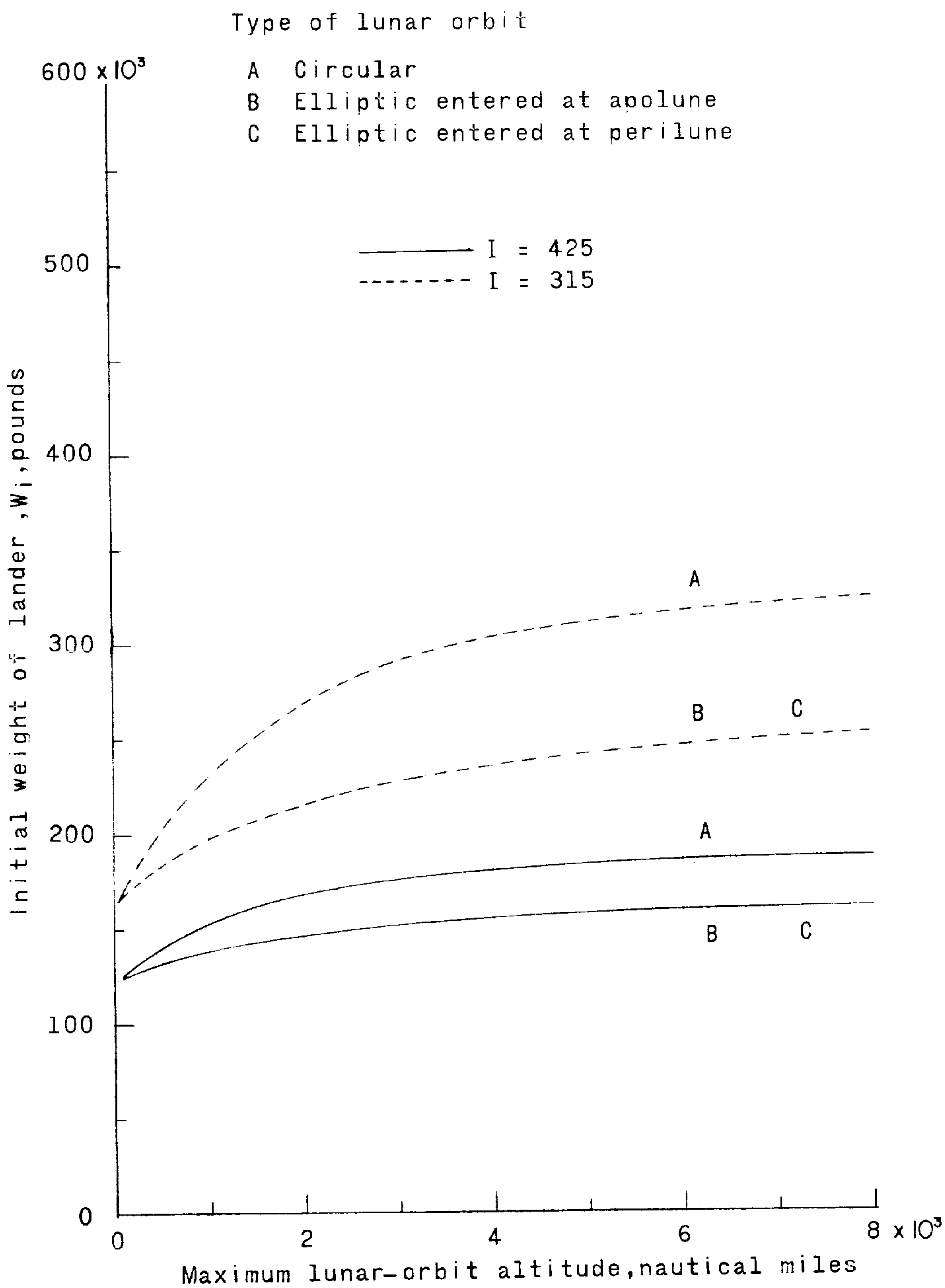


Figure 12.- Weight of lunar lander prior to descent to lunar surface as a function of maximum lunar-orbit altitude for three types of lunar orbit and two different fuels. Three-man crew; transported weight, 40,000 pounds.

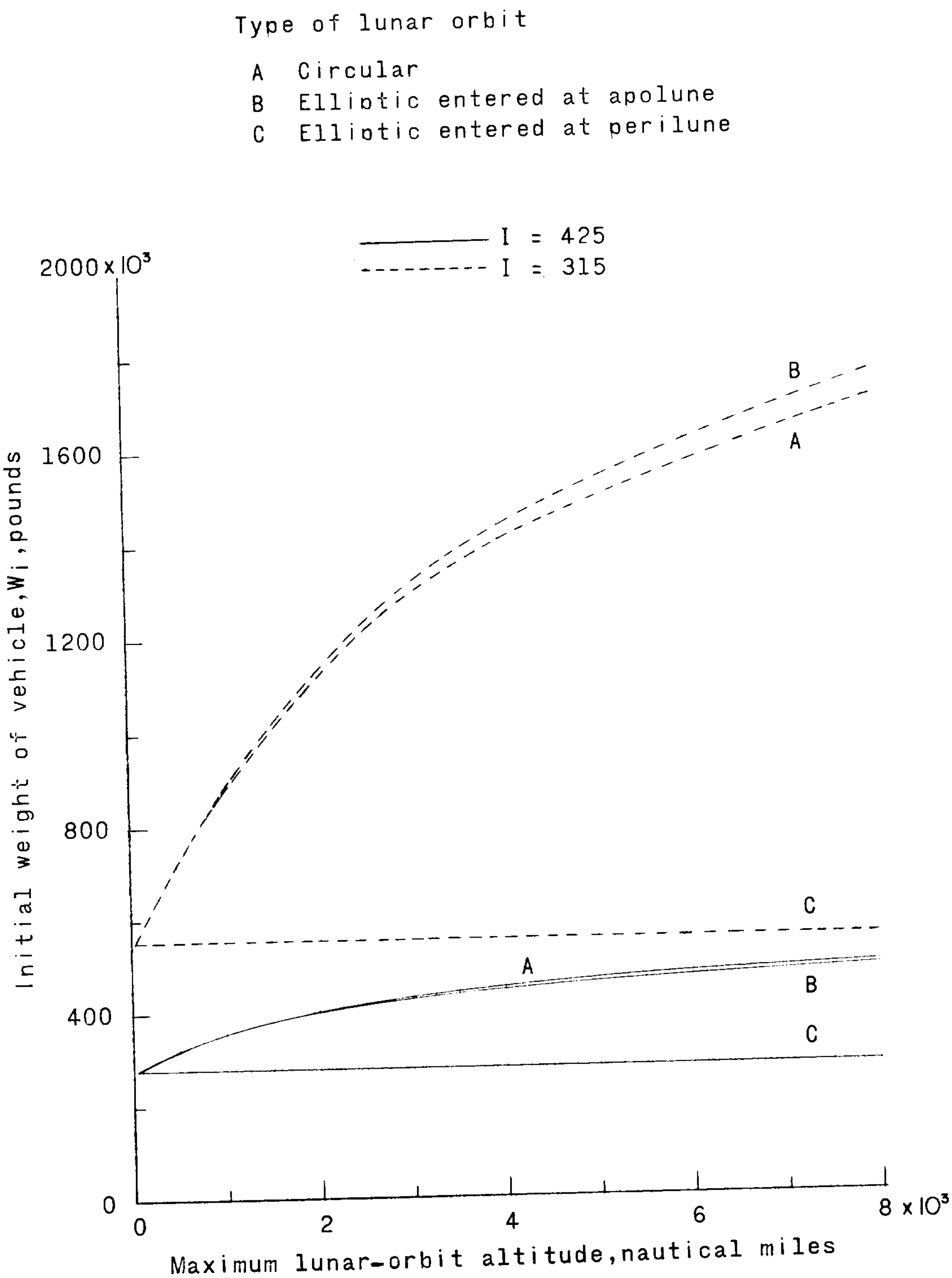


Figure 13.- Weight of direct lunar vehicle in transit to moon as a function of maximum lunar-orbit altitude for three types of lunar orbit and two different fuels. Three-man crew; transported weight, 40,000 pounds.

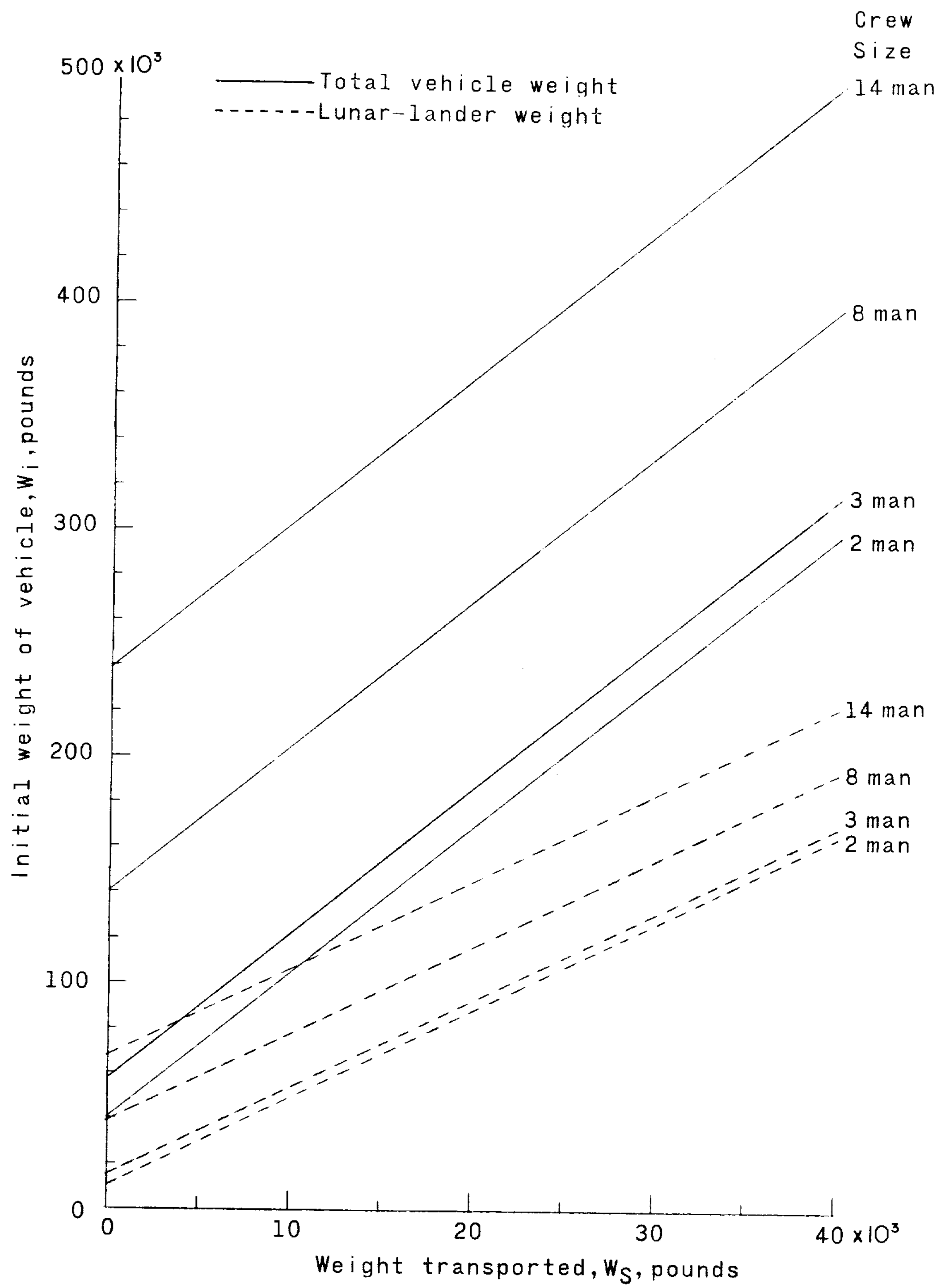


Figure 14.- Weight of lunar-orbit-rendezvous vehicle and lunar lander in transit to moon as a function of transported weight and crew size. 100-nautical-mile circular lunar orbit; I = 315 for entering and leaving lunar orbit; I = 315 for landing and take-off from moon.

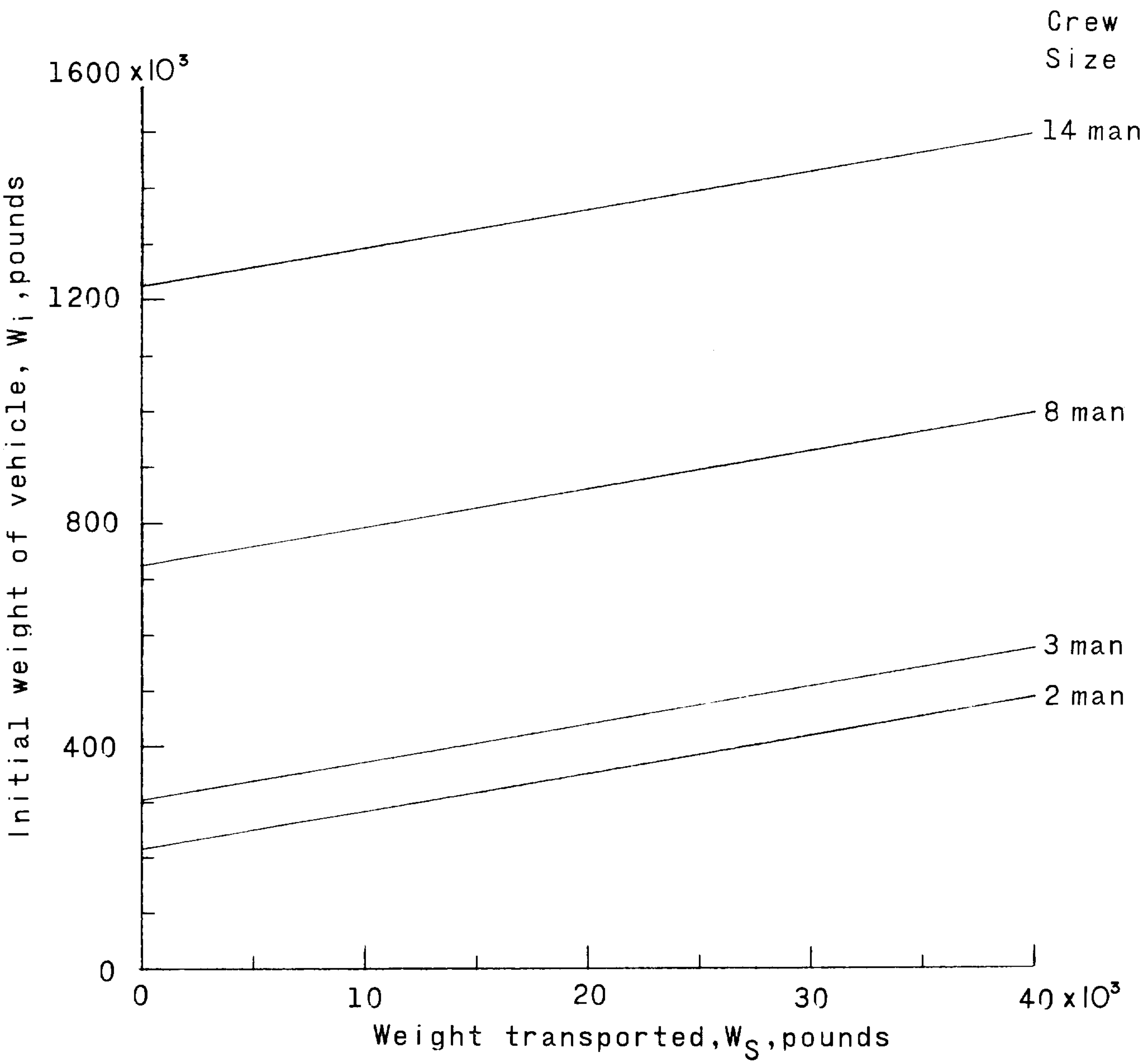


Figure 15.- Weight of direct lunar vehicle in transit to moon as a function of transported weight and crew size. 100-nautical-mile circular lunar orbit; I = 315.

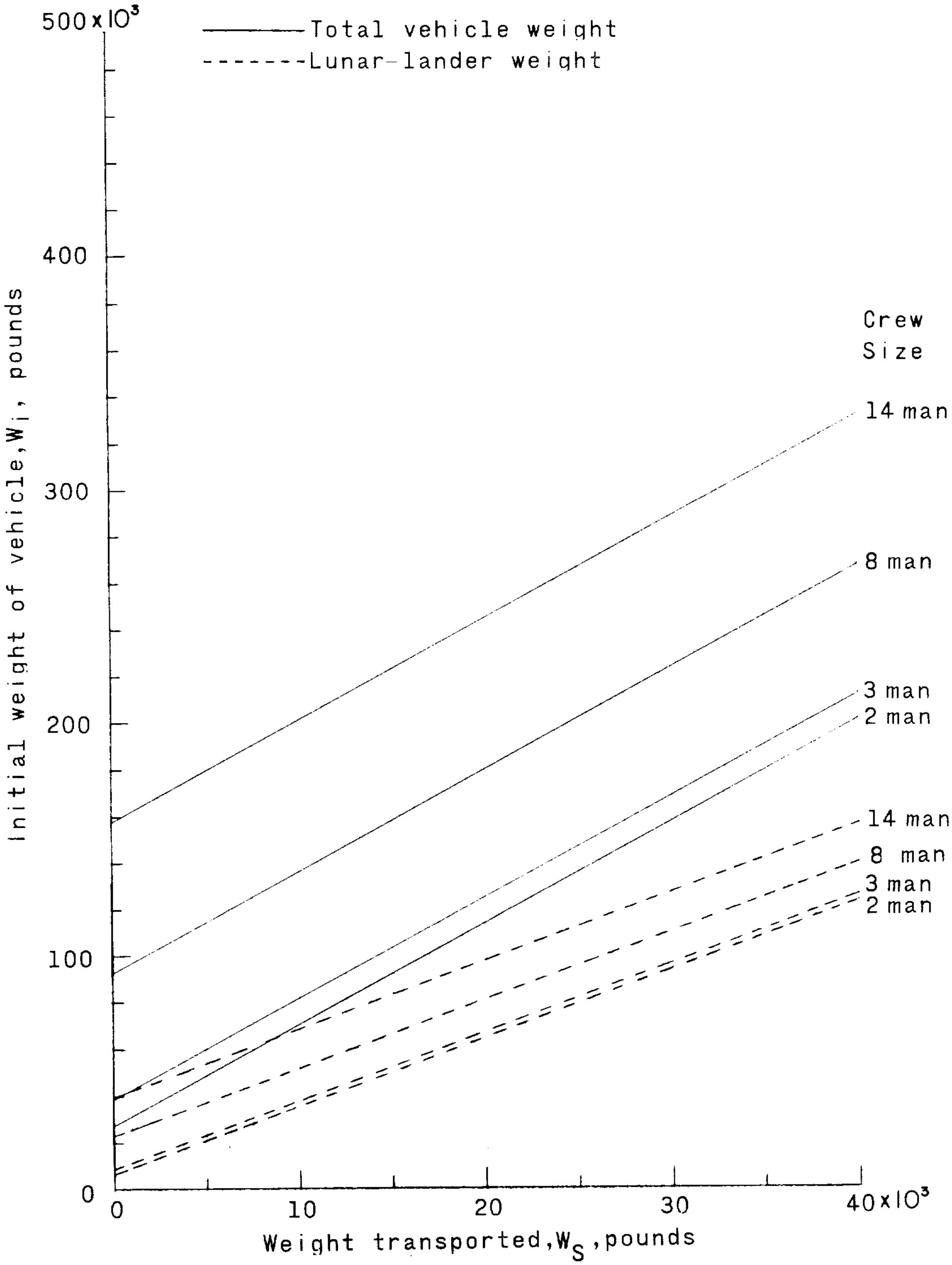


Figure 16.- Weight of lunar-orbit-rendezvous vehicle and lunar lander in transit to moon as a function of transported weight and crew size. 100-nautical-mile circular lunar orbit; I = 425 for entering and leaving lunar orbit; I = 425 for landing and take-off from moon.

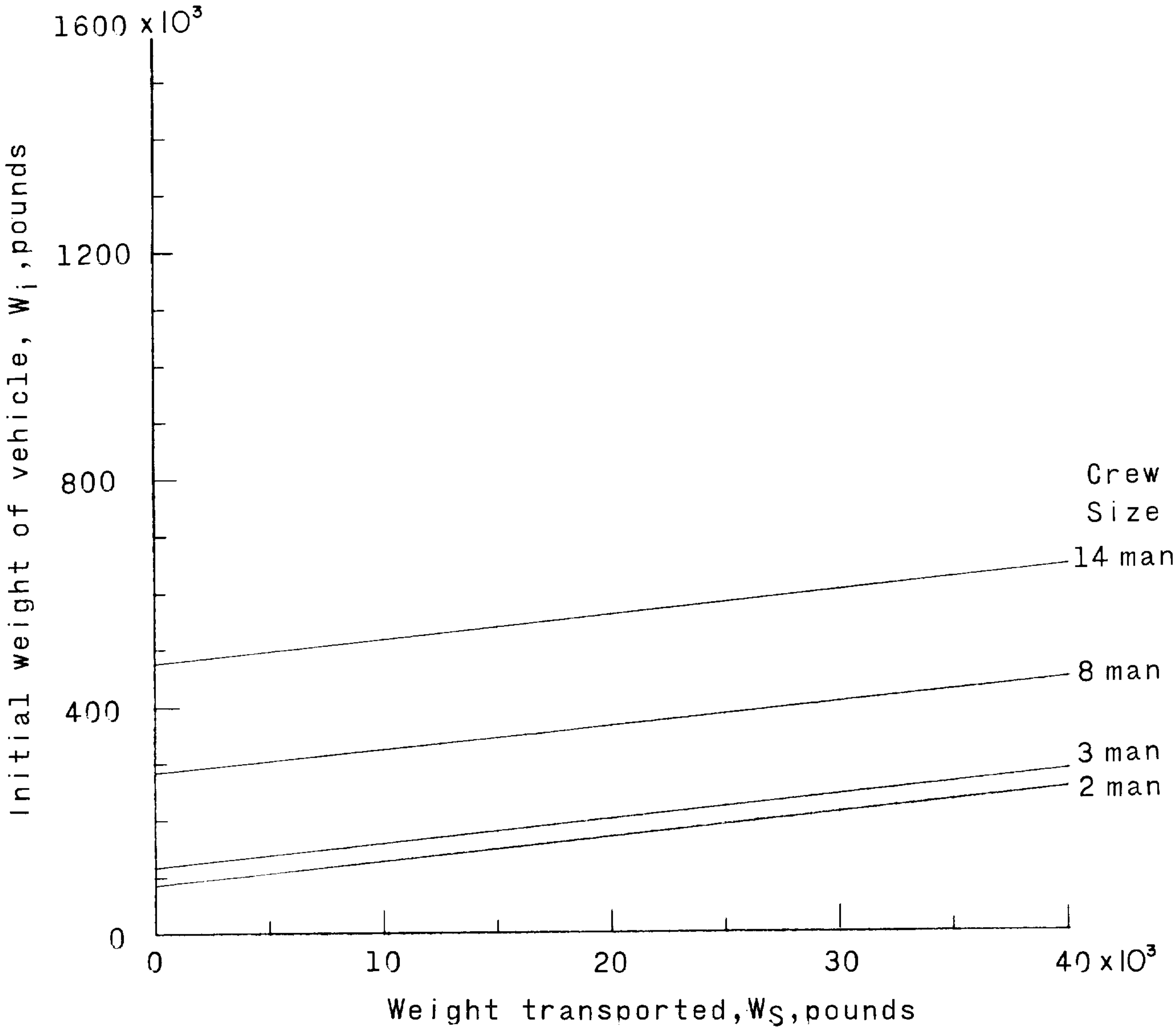


Figure 17.- Weight of direct lunar vehicle in transit to moon as a function of transported weight and crew size. 100-nautical-mile circular lunar orbit; I = 425.

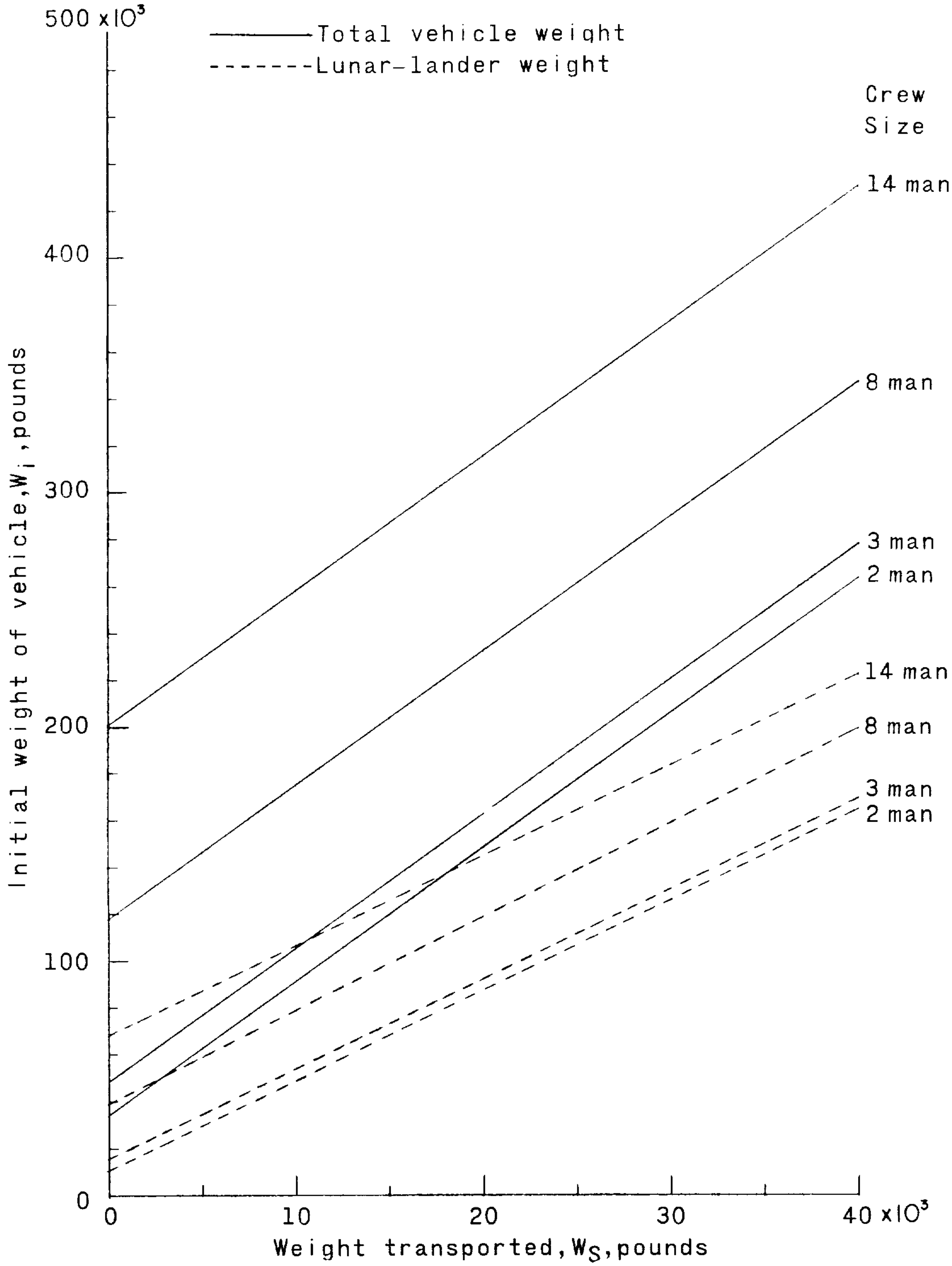


Figure 18.- Weight of lunar-orbit-rendezvous vehicle and lunar lander in transit to moon as a function of transported weight and crew size. 100-nautical-mile circular lunar orbit; I = 425 for entering and leaving lunar orbit; I = 315 for landing and take-off from moon.

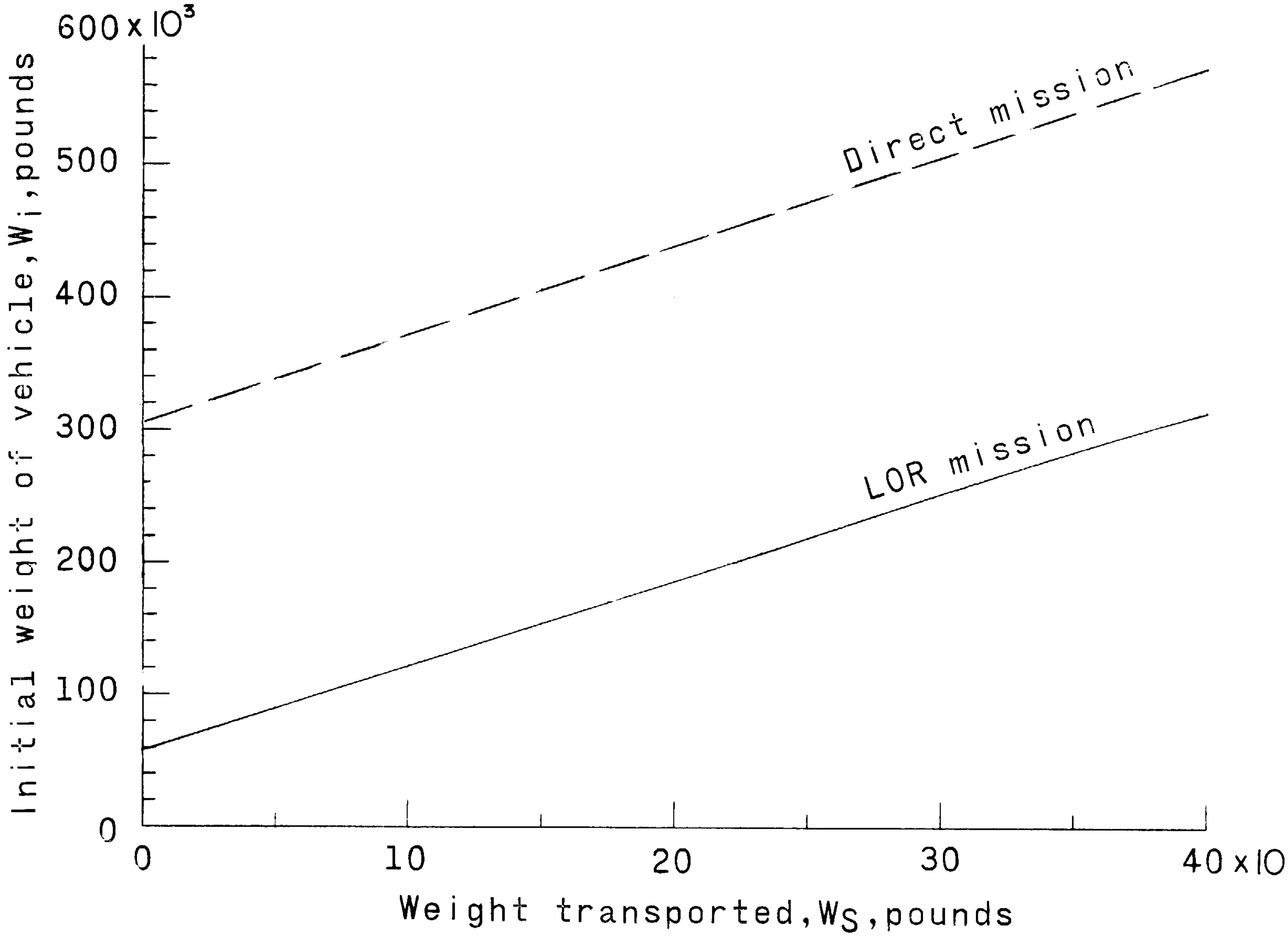


Figure 19.- Comparison of three-man vehicle weights for 100-nautical-mile circular lunar orbit with I = 315.

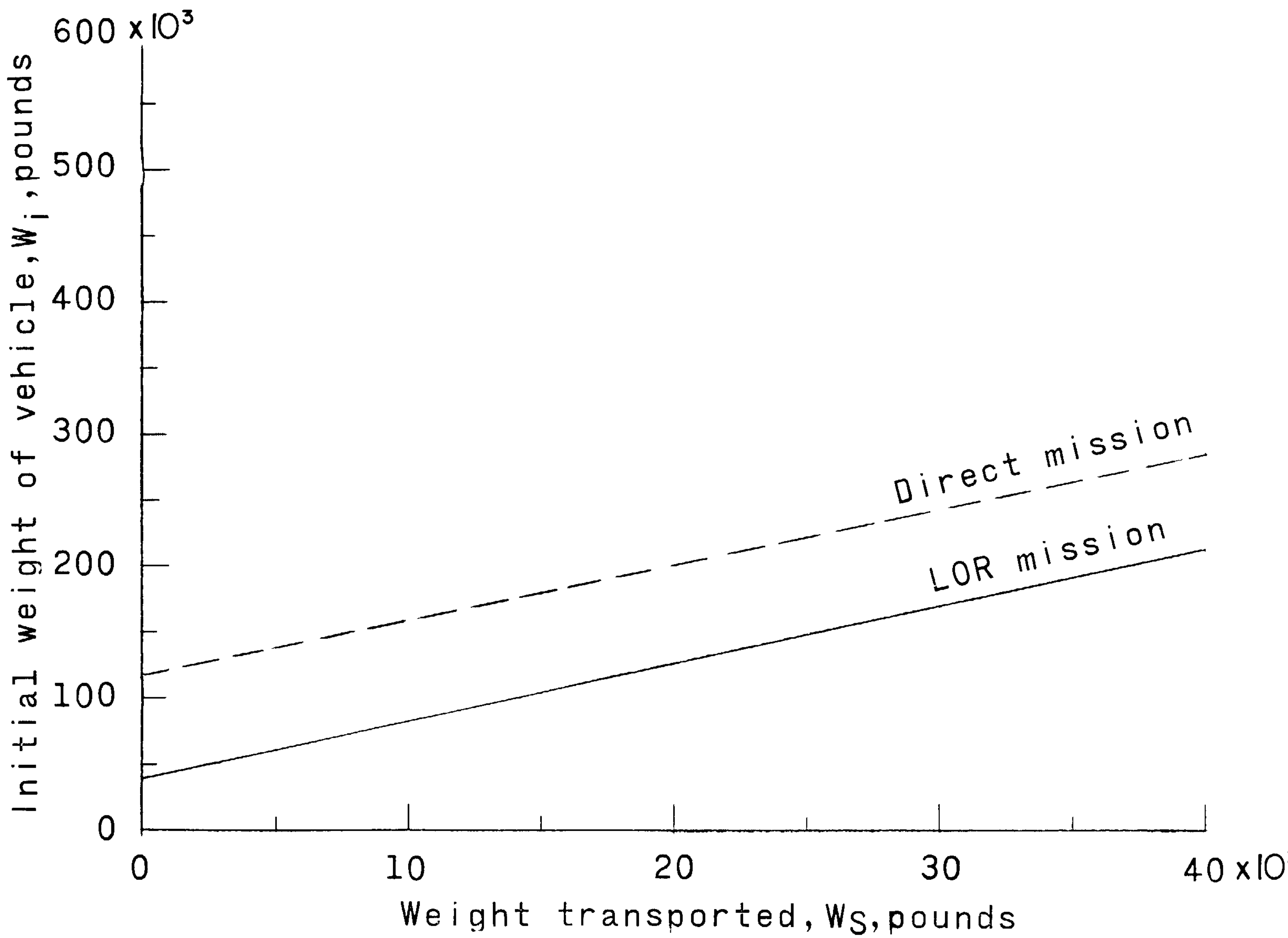


Figure 20.- Comparison of three-man vehicle weights for 100-nautical-mile circular lunar orbit with $I = 425$.

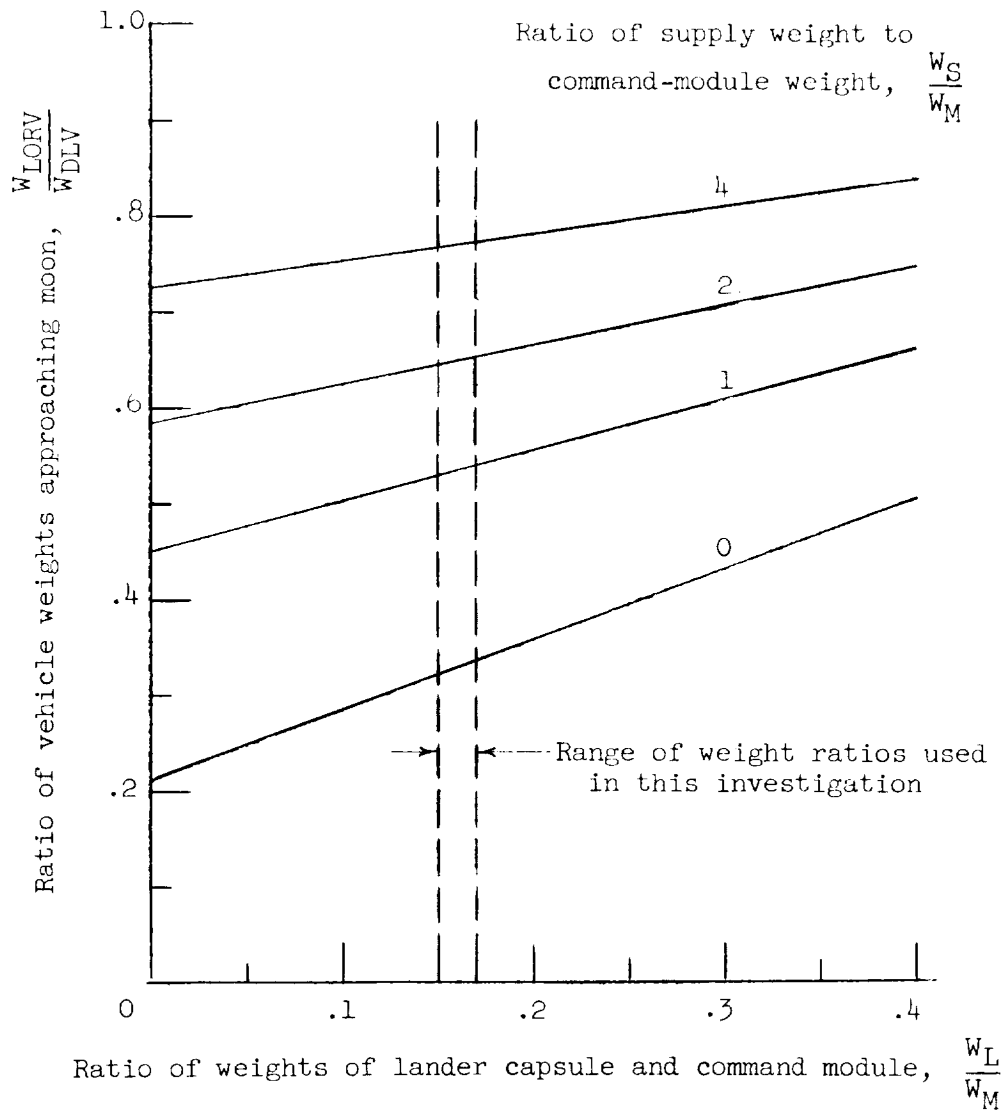


Figure 21.- Ratio of initial vehicle weights of lander mode and direct mode as a function of the ratio of weight of lander to weight of control module for various ratios of supply weight to control module weight. Three-man mission; circular orbit altitude = 100 nautical miles; I = 425.

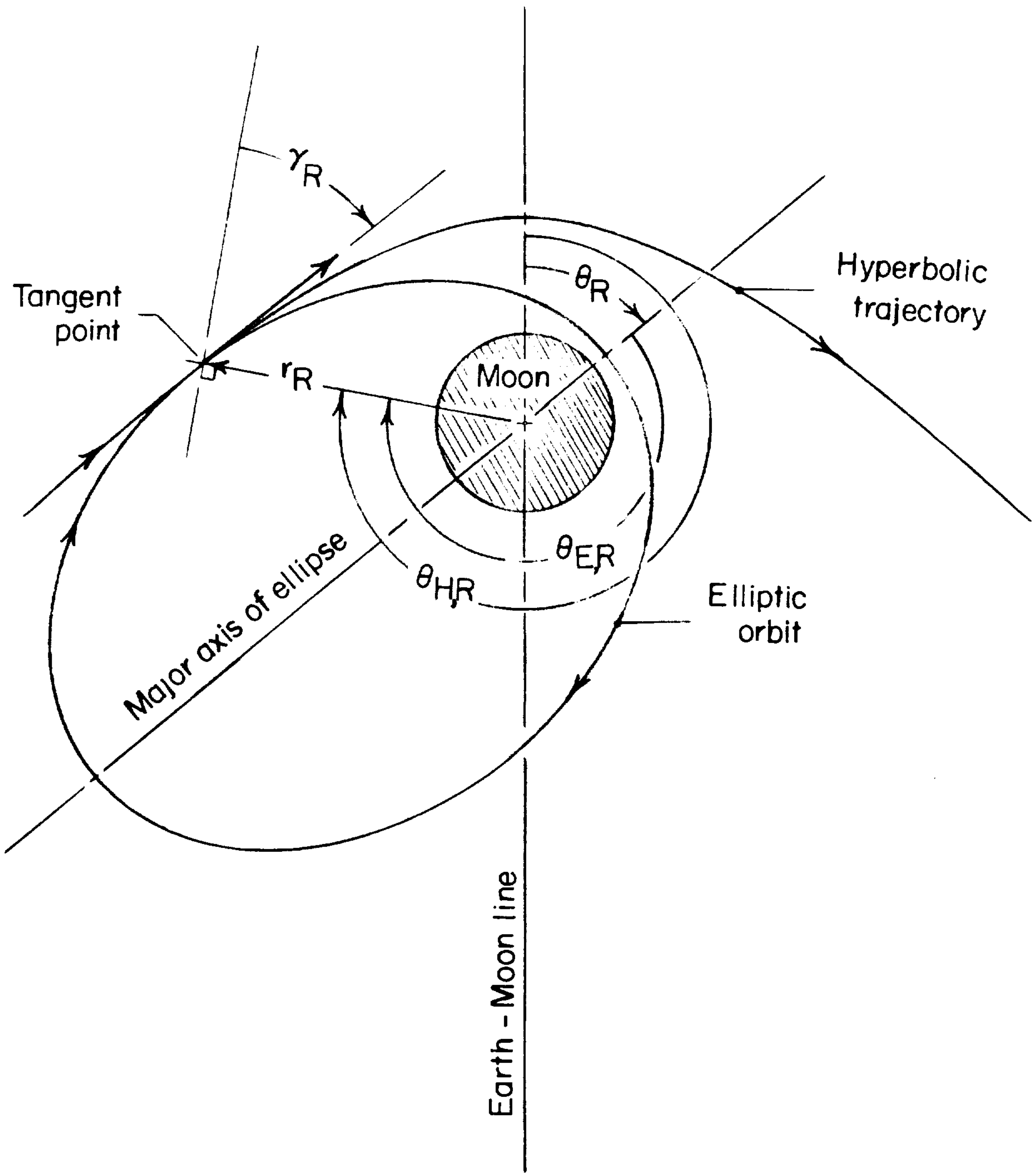


Figure 22.- Geometry of rotation of major axis of an elliptic orbit.

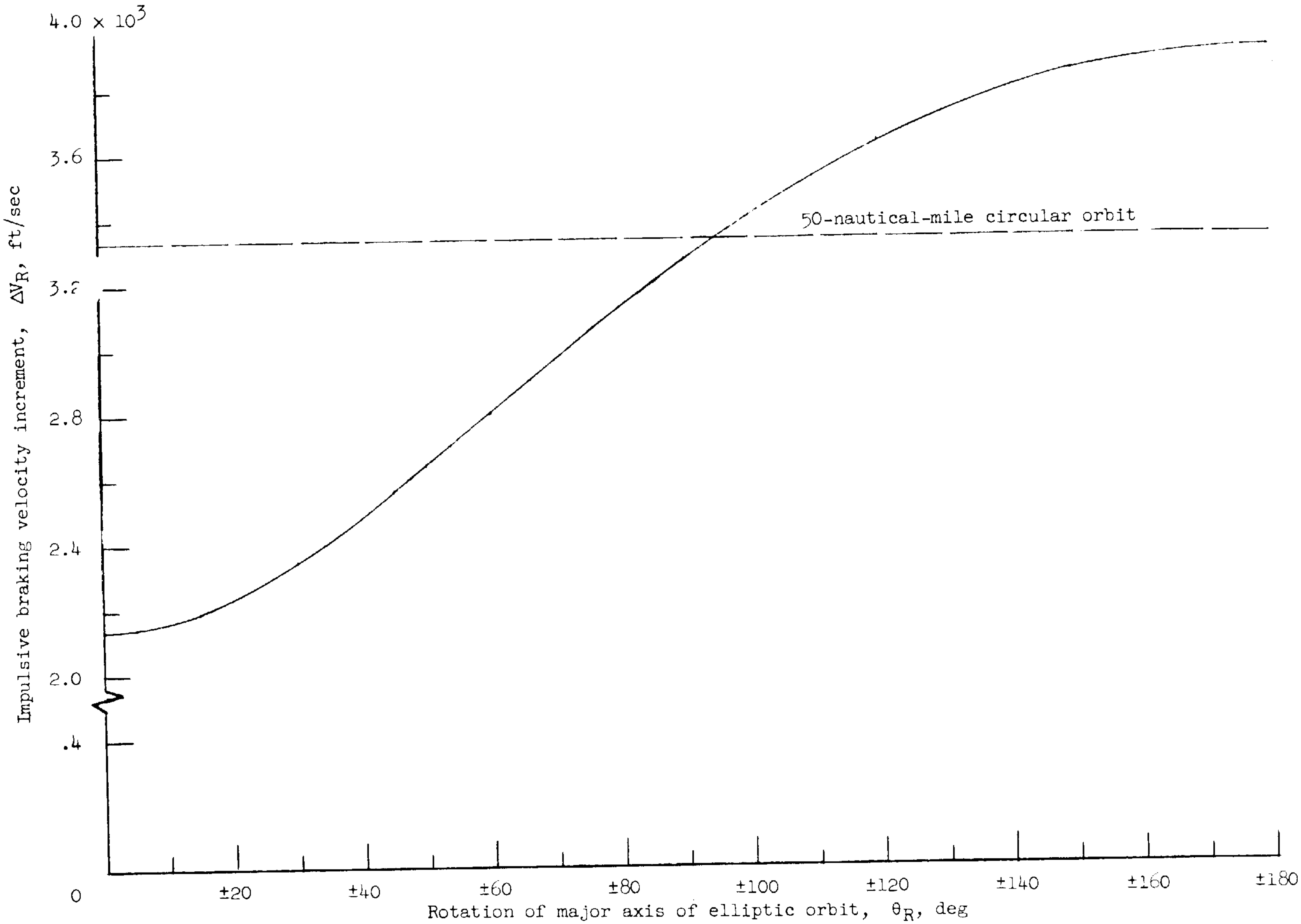


Figure 23.- Impulsive braking velocity increment as a function of rotation of the major axis of an elliptic orbit having a perilune altitude of 50 nautical miles and an apolune altitude of 2,000 nautical miles. Hyperbolic trajectory defined as having the energy level of 8,700 ft/sec at 50-nautical-mile altitude.

NASA-Langley, 1963 L-3216